Zadaci 5.5.

5.5.

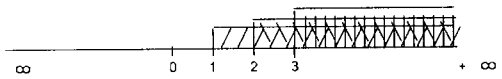
14.

1.) $\log(x-1) + \log(x-2) = 2 \cdot \log(x-3)$ prvo odredimo uvijet $x-1>0$ $x-2>0$ $x-3>0$

$x>1$ $x>2$ $x>3$

$\log\left((x-1)\cdot(x-2)\right) = \log(x-3)^2$ sada primjenimo $\log x = \log y \Rightarrow x = y$

uvjet zadatka je presjek ta tri uvjeta



pa je uvjet zadatka je $x>3$ ili $x \in \langle 3, \infty \rangle$

$(x-1)\cdot(x-2) = (x-3)^2$

$x^2-2x-x+2 = x^2-6x+9$

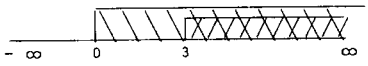
$x^2-x^2-2x-x+6x = 9-2$

$3x = 7 \quad / : 3$

$x = \frac{7}{3}$ kako uvijet kaže da x mora biti veči od tri to dobivena vrijednost $x = \frac{7}{3}$ nije rješenje ove jednadžbe . . .

dakle jednadžba nema rješenja

2.) $\log x + \log(x-3) = 1$ odredimo uvjet $x>0$ $x-3>0$ $x>3$



$x>3$ ili $x \in \langle 3, \infty \rangle$

$\log\left(x\cdot(x-3)\right) = \log 10$

$x\cdot(x-3) = 10$

$x^2-3x-10 = 0$

$x_{1,2} = \frac{-(-3)\pm\sqrt{(-3)^2-4\cdot1\cdot(-10)}}{2\cdot1} = \frac{3\pm\sqrt{9+40}}{2} = \frac{3\pm\sqrt{49}}{2} = \frac{3\pm7}{2}$

$x_1 = \frac{10}{2} = 5 \qquad x_2 = \frac{3-7}{2} = -\frac{4}{2} = -2$

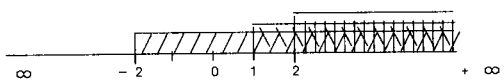
dobivene vrijednosti za x su $x_1 = 5$ i $x_2 = -2$ kako uvjet kaže da x mora biti veči od tri to je samo jedno valjano rješenje . . .

jedino rješenje je $x = 5$

3.) $\log(x-2) + \log(x+2) = 2 \cdot \log(x-1)$ prvo odredimo uvijet

$x-2>0$ $x+2>0$ $x-1>0$

$x>2$ $x>-2$ $x>1$



uvjet je $x>2$ ili $x \in \langle 2, \infty \rangle$

$\log(x-2)\cdot(x+2) = \log(x-1)^2$

$(x-2)\cdot(x+2) = (x-1)^2$

$x^2-4 = x^2-2x+1$

$x^2-x^2+2x = 4+1$

$2x = 5 \quad / : 2$

$x = \frac{5}{2}$

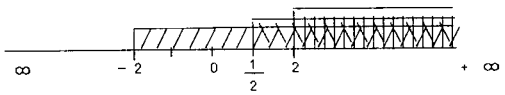
kako je $x = \frac{5}{2}$ a to je $= 2{,}5$ . . . ujvet zadatka kaže $x>2$ pa ovaj x zadovoljava uvjet i to je rješenje jed.

4.) $\log(2x-1) - \log(x+2) = \log(x-2)$ odredimo uvijet

$2x-1>0$ $x+2>0$ $x-2>0$

$2x>1 \quad / : 2$ $x>-2$ $x>2$

$x>\frac{1}{2}$



$\log\frac{2x-1}{x+2} = \log(x-2)$

$\frac{2x-1}{x+2} = x-2 \quad / \cdot(x+2)$

$2x-1 = (x-2)\cdot(x+2)$

$2x-1 = x^2-4$

$-x^2+2x+4-1 = 0$

$-x^2+2x+3 = 0 \quad / \cdot(-1)$

$x^2-2x-3 = 0$

$x_{1,2} = \frac{-(-2)\pm\sqrt{(-2)^2-4\cdot1\cdot(-3)}}{2\cdot1} = \frac{2\pm\sqrt{4+12}}{2} = \frac{2\pm\sqrt{16}}{2} = \frac{2\pm4}{2}$

$x_1 = \frac{2+4}{2} = \frac{6}{2} = 3 \qquad x_2 = \frac{2-4}{2} = -\frac{2}{2} = -1$

kako je uvijet $x>2$ to x_1 zadovoljava ali x_2 ne zadovoljava uvijet , pa je jedino rješenje $x = 3$

14.

5.5.

5.) $\log(3x-5) - \frac{1}{2}\cdot\log(x+1) = 1 - \log 5$

$\log(3x-5) - \log(x+1)^{\frac{1}{2}} = \log 10 - \log 5$

$\log(3x-5) - \log\sqrt{x+1} = \log\frac{10}{5}$

$\log\frac{3x-5}{\sqrt{x+1}} = \log 2$

$\frac{3x-5}{\sqrt{x+1}} = 2 \quad / \cdot\sqrt{x+1}$

$3x-5 = 2\cdot\sqrt{x+1} \quad /^2$

$(3x-5)^2 = 4\cdot\sqrt{(x+1)^2}$

$9x^2 - 30x + 25 = 4\cdot(x+1)$

$9x^2 - 30x + 25 = 4x + 4$

$9x^2 - 30x - 4x + 25 - 4 = 0$

$9x^2 - 34x + 21 = 0$

$x_{1,2} = \frac{-(-34) \pm \sqrt{(-34)^2 - 4\cdot 9\cdot 21}}{2\cdot 9} = \frac{34 \pm \sqrt{1156-756}}{18} = \frac{34\pm\sqrt{400}}{18} = \frac{34\pm 20}{18}$

$x_1 = \frac{34+20}{18} = \frac{54}{18} = 3 \qquad x_2 = \frac{34-20}{18} = \frac{14}{18} = \frac{7}{9}$

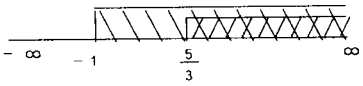
uvjet kaže $x > \frac{5}{3}$ ili $x > 1{,}67$ tako da x_2 otpada i jedino rješenje je $x = 3$

odredimo uvijet

$3x - 5 > 0 \qquad x + 1 > 0$

$3x > +5 \ /:3 \qquad x > -1$

$x > +\frac{5}{3}$



uvjet je $x > \frac{5}{3}$ ili $x \in \left\langle \frac{5}{3}, \infty \right\rangle$

6.) $\log(3x-2) - 2 = \frac{1}{2}\cdot\log(x+2) - \log 50$

$\log(3x-2) - \log 100 = \log(x+2)^{\frac{1}{2}} - \log 50$

$\log\frac{3x-2}{100} = \log\frac{\sqrt{x+2}}{50}$

$\frac{3x-2}{100} = \frac{\sqrt{x+2}}{50} \quad / \cdot 100$

$3x-2 = 2\cdot\sqrt{x+2} \quad /^2$

$(3x-2)^2 = 2^2\cdot\sqrt{(x+2)^2}$

$9x^2 - 12x + 4 = 4\cdot(x+2)$

$9x^2 - 12x + 4 = 4x + 8$

$9x^2 - 12x - 4x + 4 - 8 = 0$

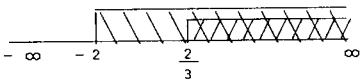
$9x^2 - 16x - 4 = 0$

odredimo uvijet

$3x - 2 > 0 \qquad x + 2 > 0$

$3x > 2 \ /:3 \qquad x > -2$

$x > \frac{2}{3}$



uvjet je $x > \frac{2}{3}$

$9x^2 - 16x - 4 = 0$

$x_{1,2} = \frac{-(-16) \pm \sqrt{(-16)^2 - 4\cdot 9\cdot(-4)}}{2\cdot 9} = \frac{16\pm\sqrt{256+144}}{18} = \frac{16\pm\sqrt{400}}{18} = \frac{16\pm 20}{18}$

$x_1 = \frac{16+20}{18} = \frac{36}{18} = 2 \qquad x_2 = \frac{16-20}{18} = -\frac{4}{18} = -\frac{2}{9}$

kako je uvijet $x > \frac{2}{3}$ to x_2 ne zadovoljava taj uvijet . . .

i jedino rješenje je $x = 2$

16.

1.) $\frac{3}{\log x - 1} = 1 + \log x$

kada smo odredili uvijet

uvodimo novu nepoznanicu

$\log x = t$

$\frac{3}{t-1} = 1 + t \quad / \cdot(t-1)$

$3 = (1+t)\cdot(t-1)$

$3 = t^2 - 1$

$-t^2 = -1 - 3$

$-t^2 = -4 \quad / \cdot(-1)$

$t^2 = 4 \quad /\sqrt{\ }$

$t_{1,2} = \pm 2$

$t_1 = 2 \qquad t_2 = -2$

$\log x = 2 \qquad \log x = -2$

$x = 10^2 \qquad x = 10^{-2}$

$x = 100 \qquad x = \frac{1}{100}$

prvo odredimo uvijet $x > 0$ i $\log x - 1 \neq 0$ → jer nazivnik nikada nesmije biti jednak nuli . . .

$\log x \neq 1$

$\log x \neq \log 10$

$x \neq 10$

dakle uvijet je $x > 0$ i $x \neq 10$ ili riječima x mora biti veći od nule i različit od 10

uvijet kaže $x \neq 10$ i $x > 0$

tako obadvije dobivene vrijednosti za x zadovoljavaju taj uvijet

pa su rješenja $x_1 = 100$ i $x_2 = \frac{1}{100} = 0{,}01$

16.

5.5.

2.) $\frac{2 \cdot \log x}{\log x - 1} - \log x = \frac{2}{\log x - 1}$ odredimo uvijet $x > 0$ i $\log x - 1 \neq 0$

$\log x \neq 1$

$\log x \neq \log 10$

$x \neq 10$

dakle uvijet je $x > 0$ i $x \neq 10$

uvedemo novu nepoznanicu

$\log x = t$

$\frac{2 \cdot t}{t-1} - t = \frac{2}{t-1} \quad / \cdot (t-1)$

$2t - t \cdot (t-1) = 2$

$2t - t^2 + t = 2$

$-t^2 + 3t - 2 = 0 \quad / \cdot (-1)$

$t^2 - 3t + 2 = 0$

$t_{1,2} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm \sqrt{1}}{2} = \frac{3 \pm 1}{2}$

$t_1 = \frac{3+1}{2} = \frac{4}{2} = 2$ $t_2 = \frac{3-1}{2} = \frac{2}{2} = 1$

$t_1 = 2$ $t_2 = 1$

$\log x = 2$ $\log x = 1$

$x = 10^2$ $x = 10^1$

$x = 100$ $x = 10$

otpada

uvijet je $x > 0$ i $x \neq 10$

tako da $x = 10$ otpada pa je jedino rješenje $x = 100$

3.) $\frac{1}{5 - \log x} + \frac{2}{1 + \log x} = 1$ odredimo uvjet $x > 0$ $5 - \log x \neq 0$ $1 + \log x \neq 0$

$-\log x \neq -5$ $\log x \neq -1$

$\log x \neq 5$

$x \neq 10^5$ $x \neq 10^{-1}$

$x \neq \frac{1}{10}$

dakle uvjet je $x > 0$, $x \neq \frac{1}{10}$ i $x \neq 10^5$

$\log x = t$

$\frac{1}{5-t} + \frac{2}{1+t} = 1 \quad / \cdot (5-t) \cdot (1+t)$

$1 \cdot (1+t) + 2 \cdot (5-t) = 1 \cdot (5-t) \cdot (1+t)$

$1 + t + 10 - 2t = 5 + 5t - t - t^2$

$t^2 + t - 2t - 5t + t + 1 + 10 - 5 = 0$

$t^2 - 5t + 6 = 0$

$t_{1,2} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1} = \frac{5 \pm \sqrt{25-24}}{2} = \frac{5 \pm \sqrt{1}}{2} = \frac{5 \pm 1}{2}$

$t_1 = \frac{5+1}{2} = \frac{6}{2} = 3$ $t_2 = \frac{5-1}{2} = \frac{4}{2} = 2$

$t_1 = 3$ $t_2 = 2$

$\log x = 3$ $\log x = 2$

$x = 10^3$ $x = 10^2$

$x = 1000$ $x = 100$

$x_1 = 1000$ $x_2 = 100$

uvjet je $x > 0$, $x \neq \frac{1}{10}$ i $x \neq 10^5$

dakle obadvije dobivene vrijednosti x zadovoljavaju uvjet . . .

4.) $\frac{1}{5 - 4 \cdot \log x} + \frac{4}{1 + \log x} = 3$ uvjet $x > 0$ $5 - 4 \cdot \log x \neq 0$ $1 + \log x \neq 0$

$-4 \cdot \log x \neq -5 \quad / : (-4)$ $\log x \neq -1$

$\log x \neq \frac{5}{4}$ $x \neq 10^{-1}$

$x \neq 10^{\frac{5}{4}}$ $x \neq \frac{1}{10}$

$x \neq \sqrt[4]{10^5}$

uvjet je $x > 0$, $x \neq \frac{1}{10}$ i $x \neq \sqrt[4]{10^5}$

$\log x = t$

$\frac{1}{5-4t} + \frac{4}{1+t} = 3 \quad / \cdot (5-4t) \cdot (1+t)$

$1 \cdot (1+t) + 4 \cdot (5-4t) = 3 \cdot (5-4t) \cdot (1+t)$

$1 + t + 20 - 16t = 3 \cdot (5 + 5t - 4t - 4t^2)$

$21 - 15t = 15 + 15t - 12t - 12t^2$

$12t^2 + 12t - 15t - 15t - 15 + 21 = 0$

$12t^2 - 18t + 6 = 0 \quad / : 6$

$2t^2 - 3t + 1 = 0$

$t_{1,2} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 2 \cdot 1}}{2 \cdot 2} = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3 \pm \sqrt{1}}{4} = \frac{3 \pm 1}{4} \rightarrow\rightarrow\rightarrow$

$\rightarrow$ $t_1 = \frac{3+1}{4} = \frac{4}{4} = 1$ $t_2 = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$

$\log x = 1$ $\log x = \frac{1}{2}$

$x = 10^1$ $x = 10^{\frac{1}{2}}$

$x_1 = 10$ $x_2 = \sqrt{10}$

obadva rješenja zadovoljavaju uvjet

17.

1) $\log^2 x + 2 \cdot \log(0{,}1x) = 1$

$\log^2 x + 2 \cdot [\log 0{,}1 + \log x] = 1$

$\log^2 x + 2 \cdot \log 0{,}1 + 2\log x - 1 = 0$

$\log^2 x + 2 \cdot (-1) + 2\log x - 1 = 0$

$\log^2 x + 2\log x - 2 - 1 = 0$

$\log^2 x + 2\log x - 3 = 0$

$t = \log x$

$t^2 + 2t - 3 = 0$

$t_{1,2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} = \frac{-2 \pm \sqrt{4+12}}{2} = \frac{-2 \pm \sqrt{16}}{2} = \frac{-2 \pm 4}{2}$

$t_1 = \frac{-2+4}{2} = 1 \qquad t_2 = \frac{-2-4}{2} = -3$

$\log x = t_1$

$\log x = 1$

Pravilo : $\log_a b = c \Rightarrow b = a^c$

$a = 10 \quad b = x \quad c = 1$

$x = 10^1$

$x = 10$

$\log x = t_2$

$\log x = -3$

$a = 10 \quad b = x \quad c = -3$

$x = 10^{-3}$

$x = \frac{1}{10^3}$

$x = \frac{1}{1000} \Rightarrow x = 0{,}001$

2) $\log(0{,}1x^2) \cdot \log \frac{10}{x} = -3$

$(\log 0{,}1 + \log x^2) \cdot (\log 10 - \log x) = -3$

$(\log 10^{-1} + 2\log x)(1 - \log x) = -3$

$(-1 \log 10 + 2\log x)(1 - \log x) = -3$

$(-1 + 2\log x)(1 - \log x) = -3$

$t = \log x$

$(-1 + 2t)(1 - t) = -3$

$-1 + t + 2t - 2t^2 = -3$

$-2t^2 + 3t - 1 + 3 = 0$

$-2t^2 + 3t + 2 = 0 \quad / \cdot (-1)$

$2t^2 - 3t - 2 = 0$

$a = 2 \quad b = -3 \quad c = -2$

$t_{1,2} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 2 \cdot (-2)}}{2 \cdot 2}$

$t_{1,2} = \frac{3 \pm \sqrt{9+16}}{4}$

$t_{1,2} = \frac{3 \pm 5}{4}$

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