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UNIVERZALNA ZBIRKA  
POTPUNO RIJEŠENIH ZADATAKA  
PRIRUČNIK ZA SAMOSTALNO UČENJE

***MATEMATIKA***

**2**

**TRIGONOMETRIJA**  
**PRAVOKUTNOG TROKUTA**

**M.I.M.-SRAGA**  
 $\sqrt{a}$

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**BESPLATNA - WEB-VARIJANTA**

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Ovo je jako skraćena varijanta naše zbirke ...

M.I.M.-SRAGA d.o.o. zadržava sva prava na reproduciranje , umnažanje , prodaju ove zbirke potpuno riješenih zadataka isključivo u okviru svog programa poduke i dopisne poduke.

**Nikakva komercijalna upotreba ove zbirke nije dozvoljena bez pismene dozvole nakladnika !**

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# Trigonometrija pravokutnog trokuta

Formule koje koristimo u rješavanju zadataka:

$$\begin{array}{lll} \sin \sphericalangle = \frac{\text{kateta nasuprot kuta}}{\text{hipotenuza}} & \sin \alpha = \frac{a}{c} & \sin \beta = \frac{b}{c} \\ \cos \sphericalangle = \frac{\text{kateta uz kut}}{\text{hipotenuza}} & \cos \alpha = \frac{b}{c} & \cos \beta = \frac{a}{c} \\ \text{tg } \sphericalangle = \frac{\text{kateta nasuprot kuta}}{\text{kateta uz kut}} & \text{tg } \alpha = \frac{a}{b} & \text{tg } \beta = \frac{b}{a} \\ \text{ctg } \sphericalangle = \frac{\text{kateta uz kut}}{\text{kateta nasuprot kuta}} & \text{ctg } \alpha = \frac{b}{a} & \text{ctg } \beta = \frac{a}{b} \end{array}$$

Izvedene formule :

$$a = c \cdot \sin \alpha = b \cdot \text{tg } \alpha = c \cdot \cos \beta = b \cdot \text{ctg } \beta = \frac{b}{\text{ctg } \alpha}$$

$$b = c \cdot \cos \alpha = c \cdot \sin \beta = a \cdot \text{tg } \beta = a \cdot \text{ctg } \alpha = \frac{a}{\text{tg } \alpha}$$

$$c = \frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{a}{\cos \beta} = \frac{b}{\cos \alpha}$$



Trigonometrijske formule za drugi razred srednje škole:

Osnovne relacije

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

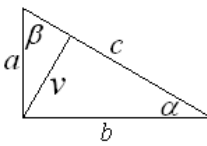
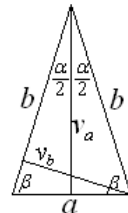
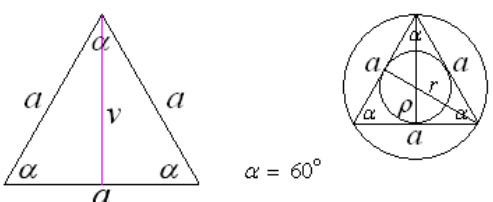
$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$1 + \operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha}$$

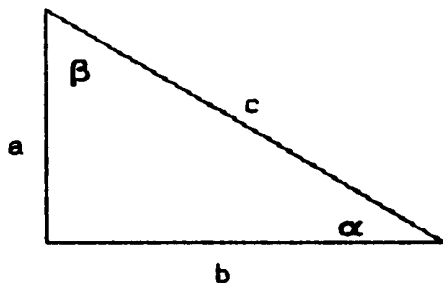
$$\operatorname{ctg} \alpha = \frac{\cos \alpha}{\sin \alpha}$$

$$1 + \operatorname{ctg}^2 \alpha = \frac{1}{\sin^2 \alpha}$$

$$\operatorname{tg} \alpha \cdot \operatorname{ctg} \alpha = 1$$

 $\sin \beta = \frac{v}{a} \quad \sin \alpha = \frac{v}{b}$ $v = a \cdot \sin \beta \quad v = b \cdot \sin \alpha$ $a = \frac{v}{\sin \beta} \quad b = \frac{v}{\sin \alpha}$	<p>Polumjer kružnice :</p> $r = \frac{c}{2} = \frac{a}{2 \cdot \sin \alpha} = \frac{b}{2 \cdot \cos \alpha} \quad (\text{opisane pravokutnom trokutu})$ $\rho = c \cdot \sin \frac{\alpha}{2} \cdot \left( \cos \frac{\alpha}{2} - \sin \frac{\alpha}{2} \right) \quad (\text{upisan pravokutnom trokutu})$
<p>Površina pravokutnog trokuta</p> $P_{\Delta} = \frac{a \cdot b}{2} = \frac{c \cdot v}{2} = \frac{a \cdot c \cdot \sin \beta}{2} = \frac{b \cdot c \cdot \sin \alpha}{2}$ $P_{\Delta} = \frac{a^2 \cdot \operatorname{tg} \beta}{2} = \frac{b^2 \cdot \operatorname{tg} \alpha}{2} = \frac{c^2 \cdot \sin 2\alpha}{4}$	<p>Jednakokraki trokut</p>  $P = \frac{a \cdot v_a}{2} \quad P = \frac{b \cdot v_b}{2} \quad o = a + 2b$ $b^2 = v_a^2 + \left(\frac{a}{2}\right)^2 \quad \frac{\alpha}{2} + \beta = 90^\circ$
<p>Istostraničan trokut</p>  $\alpha = 60^\circ$ $P = \frac{a^2 \sqrt{3}}{4} \quad o = 3 \cdot a$ $v^2 = a^2 - \left(\frac{a}{2}\right)^2 \quad v = \frac{a \sqrt{3}}{2}$ $\rho = \frac{a \sqrt{3}}{6} \quad r = \frac{a \sqrt{3}}{3}$ $\sin \alpha = \frac{v}{a}$ $v = a \cdot \sin \alpha$ $a = \frac{v}{\sin \alpha}$	<p>Iz trokuta ADB imamo:</p> $\sin \beta = \frac{v_a}{b} \quad v_a = b \cdot \sin \beta \quad b = \frac{v_a}{\sin \beta}$ $\operatorname{tg} \beta = \frac{2v_a}{a} \quad v_a = \frac{a \cdot \operatorname{tg} \beta}{2} \quad a = \frac{2v_a}{\operatorname{tg} \beta}$ $\sin \frac{\alpha}{2} = \frac{a}{2b} \quad a = 2 \cdot b \cdot \sin \frac{\alpha}{2} \quad b = \frac{a}{2 \cdot \sin \frac{\alpha}{2}}$ $\operatorname{tg} \frac{\alpha}{2} = \frac{a}{2v_a} \quad a = 2 \cdot v_a \cdot \operatorname{tg} \frac{\alpha}{2} \quad v_a = \frac{a}{2 \cdot \operatorname{tg} \frac{\alpha}{2}}$ <p>Iz trokuta BEC imamo :</p> $\sin \beta = \frac{v_b}{a} \quad v_b = a \cdot \sin \beta \quad a = \frac{v_b}{\sin \beta}$ <p>Iz trokuta AEC imamo:</p> $\sin \alpha = \frac{v_b}{b}$

# Definicije trigonometrijskih funkcija šiljastog kuta



$$\sin \alpha = \frac{a}{c} \quad \sin \beta = \frac{b}{c}$$

$$\cos \alpha = \frac{b}{c} \quad \cos \beta = \frac{a}{c}$$

$$\operatorname{tg} \alpha = \frac{a}{b} \quad \operatorname{tg} \beta = \frac{b}{a}$$

$$\operatorname{ctg} \alpha = \frac{b}{a} \quad \operatorname{ctg} \beta = \frac{a}{b}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$\operatorname{ctg} \alpha = \frac{\cos \alpha}{\sin \alpha}$$

$$\sin \sphericalangle = \frac{\text{kateta nasuprot kuta}}{\text{hipotenuza}}$$

$$\cos \sphericalangle = \frac{\text{kateta uz kut}}{\text{hipotenuza}}$$

$$\operatorname{tg} \sphericalangle = \frac{\text{kateta nasuprot kuta}}{\text{kateta uz kut}}$$

$$\operatorname{ctg} \sphericalangle = \frac{\text{kateta uz kut}}{\text{kateta nasuprot kuta}}$$





FORMULE trigonometrije za drugi razred srednje škole :

Formule za izračunavanje površine pravokutnog trokuta

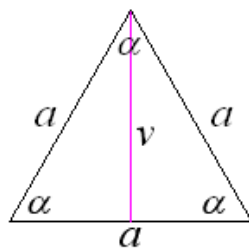
Površina pravokutnog trokuta

$$P_{\Delta} = \frac{a \cdot b}{2} = \frac{c \cdot v}{2} = \frac{a \cdot c \cdot \sin \beta}{2} = \frac{b \cdot c \cdot \sin \alpha}{2}$$

$$P_{\Delta} = \frac{a^2 \cdot \operatorname{tg} \beta}{2} = \frac{b^2 \cdot \operatorname{tg} \alpha}{2} = \frac{c^2 \cdot \sin 2\alpha}{4}$$

Formule za : istostraničan ili jednakostraničan trokut:

Istostraničan trokut



$$\alpha = 60^\circ$$



$$\sin \alpha = \frac{v}{a}$$

$$P = \frac{a^2 \sqrt{3}}{4} \quad o = 3 \cdot a$$

$$v = a \cdot \sin \alpha$$

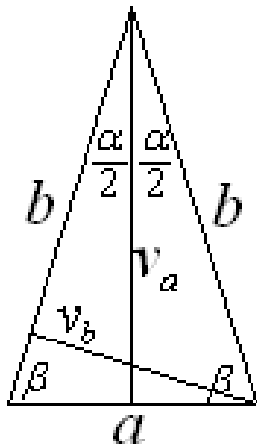
$$v^2 = a^2 - \left(\frac{a}{2}\right)^2 \quad v = \frac{a\sqrt{3}}{2}$$

$$a = \frac{v}{\sin \alpha}$$

$$\rho = \frac{a\sqrt{3}}{6} \quad r = \frac{a\sqrt{3}}{3}$$

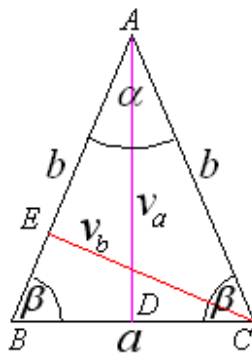


## Jednakokračan trokut



$$P = \frac{a \cdot v_a}{2} \quad P = \frac{b \cdot v_b}{2} \quad o = a + 2b$$

$$b^2 = v_a^2 + \left(\frac{a}{2}\right)^2 \quad \frac{\alpha}{2} + \beta = 90^\circ$$



Iz trokuta ADB imamo:

$$\sin \beta = \frac{v_a}{b} \quad v_a = b \cdot \sin \beta \quad b = \frac{v_a}{\sin \beta}$$

$$\operatorname{tg} \beta = \frac{2v_a}{a} \quad v_a = \frac{a \cdot \operatorname{tg} \beta}{2} \quad a = \frac{2v_a}{\operatorname{tg} \beta}$$

$$\sin \frac{\alpha}{2} = \frac{a}{2b} \quad a = 2 \cdot b \cdot \sin \frac{\alpha}{2} \quad b = \frac{a}{2 \cdot \sin \frac{\alpha}{2}}$$

$$\operatorname{tg} \frac{\alpha}{2} = \frac{a}{2v_a} \quad a = 2 \cdot v_a \cdot \operatorname{tg} \frac{\alpha}{2} \quad v_a = \frac{a}{2 \cdot \operatorname{tg} \frac{\alpha}{2}}$$

**TREKUTNO U OVOM DOKUMNETU IMAMO SAMO  
FORMULE KOJE KORISTIMO ZA RAČUNAJE PRAVOKUTNOG TROKUTA  
ALI KROZ PAR DANA NADOPUNIT ĆEMO OVAJ DOKUMNET  
SA POTPUNO RIJEŠENIM ZADACIMA IZ NAŠE PRODAJNE ZBIRKE**

**Do 22.02.2012. ovaj dokument bi morao biti kompletiran**

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**Sa napomenom da vam treba PDF sa oglednim primjerima iz TRIGONOMETRIJE-2**