**** MLADEN SRAGA ****

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UNIVERZALNA ZBIRKA
POTPUNO RIJEŠENIH ZADATAKA

PRIRUČNIK ZA SAMOSTALNO UČENJE

MATEMATIKA

2

TRIGONOMETRIJA

PRAVOKUTNOG TROKUTA

M.I.M.-SRAGA

$\sqrt{\alpha}$

Autor:
MLADEN SRAGA

Grafički urednik:
Mladen Sraga

BESPLATNA - WEB-VARIJANTA

Tisak:
M.I.M.-SRAGA d.o.o.

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Ovo je jako skraćena varijanta naše zbirke …

Trigonometrija pravokutnog trokuta

Formule koje koristimo u rješavanju zadataka:

$$\sin \sphericalangle = \frac{\text{kateta nasuprot kuta}}{\text{hipotenuza}} \qquad \sin\alpha = \frac{a}{c} \qquad \sin\beta = \frac{b}{c}$$

$$\cos \sphericalangle = \frac{\text{kateta uz kut}}{\text{hipotenuza}} \qquad \cos\alpha = \frac{b}{c} \qquad \cos\beta = \frac{a}{c}$$

$$\operatorname{tg} \sphericalangle = \frac{\text{kateta nasuprot kuta}}{\text{kateta uz kut}} \qquad \operatorname{tg}\alpha = \frac{a}{b} \qquad \operatorname{tg}\beta = \frac{b}{a}$$

$$\operatorname{ctg} \sphericalangle = \frac{\text{kateta uz kut}}{\text{kateta nasuprot kuta}} \qquad \operatorname{ctg}\alpha = \frac{b}{a} \qquad \operatorname{ctg}\beta = \frac{a}{b}$$

Izvedene formule :

$$a = c\cdot\sin\alpha = b\cdot\operatorname{tg}\alpha = c\cdot\cos\beta = b\cdot\operatorname{ctg}\beta = \frac{b}{\operatorname{ctg}\alpha}$$

$$b = c\cdot\cos\alpha = c\cdot\sin\beta = a\cdot\operatorname{tg}\beta = a\cdot\operatorname{ctg}\alpha = \frac{a}{\operatorname{tg} a}$$

$$c = \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{a}{\cos\beta} = \frac{b}{\cos\alpha}$$

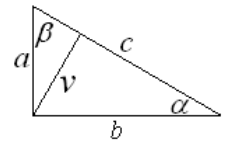
Trigonometrijske formule za drugi razred srednje škole:

Osnovne relacije

$$\sin^2\alpha+\cos^2\alpha=1 \qquad \operatorname{tg}\alpha=\frac{\sin\alpha}{\cos\alpha}$$

$$1+\operatorname{tg}^2\alpha=\frac{1}{\cos^2\alpha} \qquad \operatorname{ctg}\alpha=\frac{\cos\alpha}{\sin\alpha}$$

$$1+\operatorname{ctg}^2\alpha=\frac{1}{\sin^2\alpha} \qquad \operatorname{tg}\alpha\cdot\operatorname{ctg}\alpha=1$$



$$\sin\beta=\frac{v}{a} \qquad \sin\alpha=\frac{v}{b}$$

$$v=a\cdot\sin\beta \qquad v=b\cdot\sin\alpha$$

$$a=\frac{v}{\sin\beta} \qquad b=\frac{v}{\sin\alpha}$$

Polumjer kružnice :

$$r=\frac{c}{2}=\frac{a}{2\cdot\sin\alpha}=\frac{b}{2\cdot\cos\alpha}$$ (opisane pravokutnom trokutu)

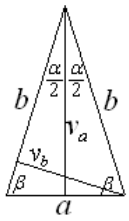
$$\rho=c\cdot\sin\frac{\alpha}{2}\cdot\left(\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}\right)$$ (upisan pravokutnom trokutu)

Površina pravokutnog trokuta

$$P_{\triangle}=\frac{a\cdot b}{2}=\frac{c\cdot v}{2}=\frac{a\cdot c\cdot\sin\beta}{2}=\frac{b\cdot c\cdot\sin\alpha}{2}$$

$$P_{\triangle}=\frac{a^2\cdot\operatorname{tg}\beta}{2}=\frac{b^2\cdot\operatorname{tg}\alpha}{2}=\frac{c^2\cdot\sin 2\alpha}{4}$$

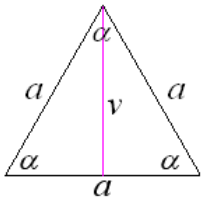
Jednakokračan trokut

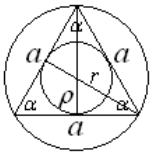


$$P=\frac{a\cdot v_a}{2} \qquad P=\frac{b\cdot v_b}{2} \qquad o=a+2b$$

$$b^2=v_a^2+\left(\frac{a}{2}\right)^2 \qquad \frac{\alpha}{2}+\beta=90^\circ$$

Istostraničan trokut





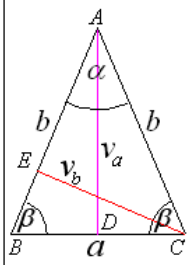
$\alpha=60^\circ$

$$\sin\alpha=\frac{v}{a} \qquad P=\frac{a^2\sqrt{3}}{4} \qquad o=3\cdot a$$

$$v=a\cdot\sin\alpha \qquad v^2=a^2-\left(\tfrac{a}{2}\right)^2 \qquad v=\frac{a\sqrt{3}}{2}$$

$$a=\frac{v}{\sin\alpha} \qquad \rho=\frac{a\sqrt{3}}{6} \qquad r=\frac{a\sqrt{3}}{3}$$

Iz trokuta ADB imamo:



$$\sin\beta=\frac{v_a}{b} \qquad v_a=b\cdot\sin\beta \qquad b=\frac{v_a}{\sin\beta}$$

$$\operatorname{tg}\beta=\frac{2v_a}{a} \qquad v_a=\frac{a\cdot\operatorname{tg}\beta}{2} \qquad a=\frac{2v_a}{\operatorname{tg}\beta}$$

$$\sin\frac{\alpha}{2}=\frac{a}{2b} \qquad a=2\cdot b\cdot\sin\frac{\alpha}{2} \qquad b=\frac{a}{2\cdot\sin\frac{\alpha}{2}}$$

$$\operatorname{tg}\frac{\alpha}{2}=\frac{a}{2v_a} \qquad a=2\cdot v_a\cdot\operatorname{tg}\frac{\alpha}{2} \qquad v_a=\frac{a}{2\cdot\operatorname{tg}\frac{\alpha}{2}}$$

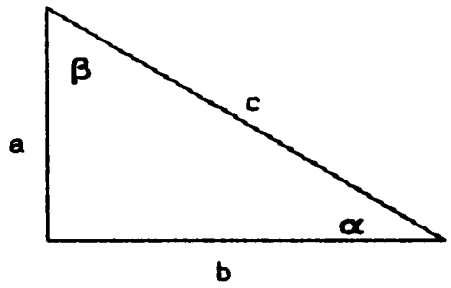
Iz trokuta BEC imamo :

$$\sin\beta=\frac{v_b}{a} \qquad v_b=a\cdot\sin\beta \qquad a=\frac{v_b}{\sin\beta}$$

Iz trokuta AEC imamo:

$$\sin\alpha=\frac{v_b}{b}$$

Definicije trigonometrijskih funkcija šiljastog kuta



$$\sin\alpha = \frac{a}{c} \qquad \sin\beta = \frac{b}{c}$$

$$\cos\alpha = \frac{b}{c} \qquad \cos\beta = \frac{a}{c}$$

$$\operatorname{tg}\alpha = \frac{a}{b} \qquad \operatorname{tg}\beta = \frac{b}{a}$$

$$\operatorname{ctg}\alpha = \frac{b}{a} \qquad \operatorname{ctg}\beta = \frac{a}{b}$$

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$tg\ \alpha = \frac{\sin\alpha}{\operatorname{cis}\alpha}$$

$$ctg\ \alpha = \frac{\cos\alpha}{\sin\alpha}$$

$$\sin\sphericalangle = \frac{\text{kateta nasuprot kuta}}{\text{hipotenuza}}$$

$$\cos\sphericalangle = \frac{\text{kateta uz kut}}{\text{hipotenuza}}$$

$$\operatorname{tg}\sphericalangle = \frac{\text{kateta nasuprot kuta}}{\text{kateta uz kut}}$$

$$\operatorname{ctg}\sphericalangle = \frac{\text{kateta uz kut}}{\text{kateta nasuprot kuta}}$$

FORMULE trigonometrije za drugi razred srednje škole :

Formule za izračunavanje površine pravokutnog trokuta

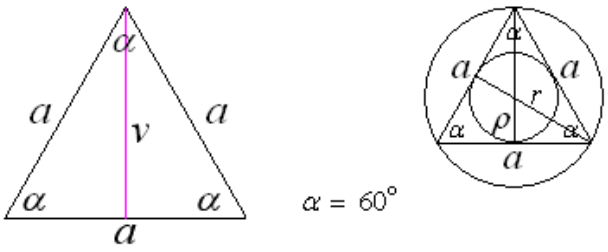
Površina pravokutnog trokuta

$$P_{\triangle} = \frac{a \cdot b}{2} = \frac{c \cdot v}{2} = \frac{a \cdot c \cdot \sin\beta}{2} = \frac{b \cdot c \cdot \sin\alpha}{2}$$

$$P_{\triangle} = \frac{a^2 \cdot \operatorname{tg}\beta}{2} = \frac{b^2 \cdot \operatorname{tg}\alpha}{2} = \frac{c^2 \cdot \sin 2\alpha}{4}$$

Formule za : istostraničan ili jednakostraničan trokut:

Istostraničan trokut

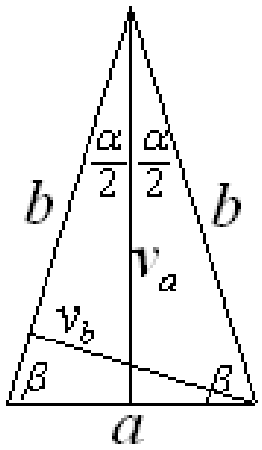


$$\sin\alpha = \frac{v}{a} \qquad P = \frac{a^2\sqrt{3}}{4} \qquad o = 3 \cdot a$$

$$v = a \cdot \sin\alpha \qquad v^2 = a^2 - \left(\tfrac{a}{2}\right)^2 \qquad v = \frac{a\sqrt{3}}{2}$$

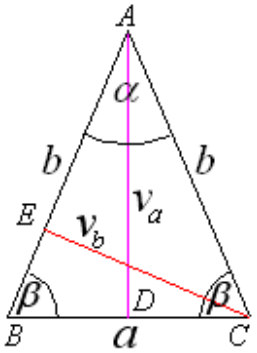
$$a = \frac{v}{\sin\alpha} \qquad \rho = \frac{a\sqrt{3}}{6} \qquad r = \frac{a\sqrt{3}}{3}$$

Jednakokračan trokut



$$P = \frac{a \cdot v_a}{2} \qquad P = \frac{b \cdot v_b}{2} \qquad o = a + 2b$$

$$b^2 = v_a^2 + \left(\frac{a}{2}\right)^2 \qquad \frac{\alpha}{2} + \beta = 90^\circ$$



Iz trokuta ADB imamo:

$$\sin\beta = \frac{v_a}{b} \qquad v_a = b \cdot \sin\beta \qquad b = \frac{v_a}{\sin\beta}$$

$$\operatorname{tg}\beta = \frac{2v_a}{a} \qquad v_a = \frac{a \cdot \operatorname{tg}\beta}{2} \qquad a = \frac{2v_a}{\operatorname{tg}\beta}$$

$$\sin\frac{\alpha}{2} = \frac{a}{2b} \qquad a = 2 \cdot b \cdot \sin\frac{\alpha}{2} \qquad b = \frac{a}{2 \cdot \sin\frac{\alpha}{2}}$$

$$\operatorname{tg}\frac{\alpha}{2} = \frac{a}{2v_a} \qquad a = 2 \cdot v_a \cdot \operatorname{tg}\frac{\alpha}{2} \qquad v_a = \frac{a}{2 \cdot \operatorname{tg}\frac{\alpha}{2}}$$

TRENUTNO U OVOM DOKUMNETU IMAMO SAMO
FORMULE KOJE KORISTIMO ZA RAČUNAJE PRAVOKUTNOG TROKUTA
ALI KROZ PAR DANA NADOPUNIT ĆEMO OVAJ DOKUMNET
SA POTPUNO RIJEŠENIM ZADACIMA IZ NAŠE PRODAJNE ZBIRKE

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