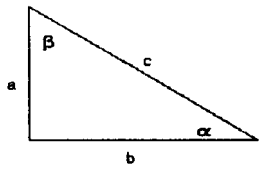
4. Trigonometrija pravokutnog trokuta

4.1. Definicije trigonometrijskih funkcija šiljastog kuta



$\sin\alpha = \frac{a}{c}$ $\quad tg\ \alpha = \frac{a}{b}$ $\quad \sin^2\alpha + \cos^2\alpha = 1$

$\cos\alpha = \frac{b}{c}$ $\quad ctg\ \alpha = \frac{b}{a}$ $\quad tg\ \alpha = \frac{\sin\alpha}{\cos\alpha}$ $\quad ctg\ \alpha = \frac{\cos\alpha}{\sin\alpha}$

1. 1) $a = 4cm$, $c = 9cm$

$\sin\alpha = ?$ $\quad \sin\beta = ?$
$\cos\alpha = ?$ $\quad \cos\beta = ?$
$tg\alpha = ?$ $\quad tg\beta = ?$
$ctg\alpha = ?$ $\quad ctg\beta = ?$

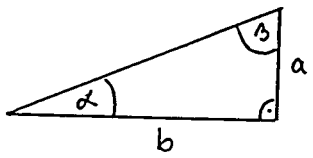
$\sin\alpha = \frac{a}{c} = \frac{4}{9}$ $\quad \sin\beta = \frac{b}{c} = \frac{\sqrt{65}}{9}$

$\cos\alpha = \frac{b}{c} = \frac{\sqrt{65}}{9}$ $\quad \cos\beta = \frac{a}{c} = \frac{4}{9}$

$tg\alpha = \frac{a}{b} = \frac{4}{\sqrt{65}}$ $\quad tg\beta = \frac{b}{a} = \frac{\sqrt{65}}{4}$

$ctg\alpha = \frac{b}{a} = \frac{\sqrt{65}}{4}$ $\quad ctg\beta = \frac{a}{b} = \frac{4}{\sqrt{65}}$

$c^2 = a^2 + b^2$
$b = \sqrt{c^2 - a^2}$
$b = \sqrt{9^2 - 4^2}$
$b = \sqrt{81 - 16}$
$b = \sqrt{65}$
$b = 8{,}062$



4.1.

1.

2) $a = 6cm$
$b = 9cm$
$\alpha = ?$
$\beta = ?$

$c^2 = a^2 + b^2$
$c = \sqrt{a^2+b^2}$
$c = \sqrt{6^2+9^2}$
$c = \sqrt{36+81}$
$c = \sqrt{117}$
$c = 10{,}82$

$\sin\alpha = \frac{a}{c} = \frac{6}{\sqrt{117}}$ $\quad \sin\beta = \frac{b}{c} = \frac{9}{\sqrt{117}}$

$\cos\alpha = \frac{b}{c} = \frac{9}{\sqrt{117}}$ $\quad \cos\beta = \frac{a}{c} = \frac{6}{\sqrt{117}}$

$tg\,\alpha = \frac{a}{b} = \frac{6}{9} = \frac{2}{3}$ $\quad tg\,\beta = \frac{b}{a} = \frac{9}{6} = \frac{3}{2}$

$ctg\,\alpha = \frac{b}{a} = \frac{9}{6} = \frac{3}{2}$ $\quad ctg\,\beta = \frac{a}{b} = \frac{6}{9} = \frac{2}{3}$

3) $b = 4cm$
$c = 8cm$

$c^2 = a^2 + b^2$
$a^2 = c^2 - b^2$
$a = \sqrt{c^2-b^2}$
$a = \sqrt{8^2-4^2}$
$a = \sqrt{64-16}$
$a = \sqrt{48}$
$a \approx 6{,}93$

$\sin\alpha = \frac{a}{c} = \frac{\sqrt{48}}{8}$ $\quad \sin\beta = \frac{b}{c} = \frac{4}{8} = \frac{1}{2}$

$\cos\alpha = \frac{b}{c} = \frac{4}{8} = \frac{1}{2}$ $\quad \cos\beta = \frac{a}{c} = \frac{\sqrt{48}}{8}$

$tg\,\alpha = \frac{a}{b} = \frac{\sqrt{48}}{4}$ $\quad tg\,\beta = \frac{b}{a} = \frac{4}{\sqrt{48}}$

$ctg\,\alpha = \frac{b}{a} = \frac{4}{\sqrt{48}}$ $\quad ctg\,\beta = \frac{a}{b} = \frac{\sqrt{48}}{4}$

4) $a = 15cm$
$b = 9cm$

$c^2 = a^2 + b^2$
$c = \sqrt{a^2+b^2}$
$c = \sqrt{15^2+9^2}$
$c = \sqrt{225+81}$
$c = \sqrt{306}$
$c \approx 17{,}49$

$\sin\alpha = \frac{a}{c} = \frac{15}{\sqrt{306}}$ $\quad \sin\beta = \frac{b}{c} = \frac{9}{\sqrt{306}}$

$\cos\alpha = \frac{b}{c} = \frac{9}{\sqrt{306}}$ $\quad \cos\beta = \frac{a}{c} = \frac{15}{\sqrt{306}}$

$tg\,\alpha = \frac{a}{b} = \frac{15}{9} = \frac{5}{3}$ $\quad tg\,\beta = \frac{b}{a} = \frac{9}{15} = \frac{3}{5}$

$ctg\,\alpha = \frac{b}{a} = \frac{9}{15} = \frac{3}{5}$ $\quad ctg\,\beta = \frac{a}{b} = \frac{15}{9} = \frac{5}{3}$

specijal-"BOS-ov"cjenik 2017.

20-60% POPUSTA NA ZBIRKE IZ TABLICE

PO DIJELOVIMA ili po POGLAVLJIMA

ovi POPUSTI vrijede od 02.01.2017. do 21.03.2017.g.

Br.	Naziv ZBIRKE riješenih zadataka	Puna cijena	02.01.17.-11.02.17. 30-60%	12.02.17.-21.03.17. 15-40%
	SREDNJA ŠKOLA			
1.	**Matematika-1-** po **Dakić-Elezović**	740 kn	**333**kn	**435** kn
1.A	ili samo **I –** polugodište (I ,II, III poglavlje)	335 kn	**159** kn	**220** kn
1.E	ili samo NEJEDNADŽBE … 2.8.—4.4..	**200 kn**	**99 kn**	**130 kn**
1.B	**II –** polugodište (IV, V, VII, VIII poglavlje)	465 kn	**199** kn	**325** kn
9.	**Testovi po Dakiću Mat-1**- komplet rješenja - **Zbirke zadataka s pismenih ispita** – Mat-1	555 kn	**315** kn	**360** kn
9.A	ili samo **I** – polugodište (I - II grupa)	200 kn	**111** kn	**150** kn
9.B	**II-** polugodište (III, V, VI, VII grupa)	355 kn	**199** kn	**266** kn
11.	**Matematika -2- - Dakić-Elezović**			
11.A	ili samo **I -** polugodište (I- IV poglavlja)	450 kn	**199** kn	**250** kn
11.B	**II -** polugodište (V - VII poglavlja)	450 kn	**229** kn	**295** kn
	Matematika -2- Dakić-Elezović po dijelovima:			
11.C	**KOMPLEKSNI BROJEVI**	100 kn	**60** kn	**80** kn
11.D	**KVADRATNA JEDNADŽBA**	120 kn	**80** kn	**90** kn
11.E	**POLINOMI 2. STUPNJA**	120 kn	**80** kn	**90** kn
11.F	**TRIGONOMETRIJA pravokutnog trokuta**	140 kn	**80** kn	**90 kn**
11.G	**Eksponencijalne i logaritamske funkcije**	150 kn	**90** kn	**100 kn**
12.	**Fizika-2-** po Mikuličić,Varičak,Vernić (toplina i elektricitet)	240 kn	**140** kn	**155** kn
13.	**Testovi po Dakiću Mat-2**- (I-VI grupa) rješenja - **Zbirke zadataka s pismenih ispita** – Mat-2	540 kn	**310** kn	**351** kn
13.A	ili samo **I** – polugodište (I-IV grupa)	340 kn	**199** kn	**238** kn
13.B	**II** – polugodište (V-VI grupa)	200 kn	**150** kn	**170** kn
	ili svako poglavlje – za sebe	155 kn	**50 kn**	**79 kn**
14.	**Matematika-3 - Dakić-Elezović**			
14.A	ili samo **I -** polugodište (trigonometrija)	410 kn	**199** kn	**270** kn
14.B	**II-**polugodište (vektori i analitička geometrija)	440 kn	**239** kn	**289** kn

	Matematika-3 Dakić-Elezović - po **DIJELOVIMA**			
	1. polugodište			
14.A-1	**Trigonometrija 1. dio –** svi zadaci od. 1.1.- 3.3.	210 kn	**99 kn**	**140 kn**
14.A-2	**Trigonometrija 2. dio –** svi zadaci od. 3.3.- 6.5.	210 kn	**99 kn**	**140 kn**
	2. polugodište			
14.C	**VEKTORI –** svi zadaci od. 7.1. – do 7.7.	120 kn	**80** kn	**90** kn

Kod narudžbe matematičkih zbirki riješenih zadataka bitn

Za sve
Ostale zbirke potpuno riješenih zadataka koje vam trebaju kontaktirajte nas mailo mili putem telefona …
-
ZA SVE INFORMACIJE I NARUDŽBE MOŽETE SE OBRATITI SVAKI DAN OD $09^{00} - 20^{00}$ ili mailom od 0-do-24
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OVO JE SAMO JEDAN MANJI DIO OGLEDNIH PRIMJERA POTPUNO RIJEŠENIH ZADATAKA PO ŠKOLSKOJ ZBIRCI DAKIĆ-ELEZOVIĆ

U NASTAVKU
IMATE
20 stranica
sa detaljno riješenim zadacima
uz većinu tih zadataka
imamo i prateće video instrukcije i upute
pogled na neke video
instrukcije
imate ovdje link >>>>

2.

Pravokutan trokut

$a = 5$
$b = 12$

$\sin\alpha = ?$
$\cos\alpha = ?$
$tg\,\alpha = ?$
$ctg\,\alpha = ?$

hipotenuza c:

$c^2 = a^2 + b^2$
$c^2 = 5^2 + 12^2$
$c^2 = 25 + 144$
$c = \sqrt{169}$
$c = 13$

$\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{5}{13}$

$\cos\alpha = \frac{b}{c}$
$\cos\alpha = \frac{12}{13}$

$tg\,\alpha = \frac{a}{b}$
$tg\,\alpha = \frac{5}{12}$

$ctg\,\alpha = \frac{b}{a}$
$ctg\,\alpha = \frac{12}{5}$

3.

$a = 7$ cm
$c = 25$ cm

tg i ctg = ?

1. $b^2 = c^2 - a^2$
   $b^2 = 25^2 - 7^2$
   $b^2 = 576 \;/\sqrt{\ }$
   $b = 24$ cm

2. $tg\,\alpha = \frac{a}{b}$
   $tg\,\alpha = \frac{7}{24}$

3. $ctg\,\alpha = \frac{b}{a}$
   $ctg\,\alpha = \frac{24}{7}$

4. $tg\,\beta = \frac{b}{a}$
   $tg\,\beta = \frac{24}{7}$

5. $ctg\,\beta = \frac{a}{b}$
   $ctg\,\beta = \frac{7}{24}$

8. Pravokutan trokut:

1) $a = 4cm$, $\sin\alpha = \frac{2}{3}$, $c = ?$

$\sin\alpha = \frac{a}{c}$

$\frac{2}{3} = \frac{4}{c} \;/\cdot 3c$

$2c = 4\cdot 3 \;/:2$

$c = \frac{12}{2}$

$c = 6cm$

2) $a = 4cm$, $tg\,\alpha = \frac{2}{3}$, $b = ?$

$tg\,\alpha = \frac{a}{b}$

$\frac{2}{3} = \frac{4}{b} \;/\cdot 3b$

$2b = 4\cdot 3$

$2b = 12 \;/:2$

$b = 6cm$

3) $c = 4cm$, $\sin\alpha = \frac{2}{3}$, $a = ?$

$\sin\alpha = \frac{a}{c}$

$\frac{2}{3} = \frac{a}{4} \;/\cdot 12$

$4\cdot 2 = 3\cdot a$

$3a = 8 \;/:3$

$a = \frac{8}{3}\,cm$

4) $c = 4cm$, $\cos\alpha = \frac{2}{3}$, $b = ?$

$\cos\alpha = \frac{b}{c}$

$\frac{2}{3} = \frac{b}{4} \;/\cdot 12$

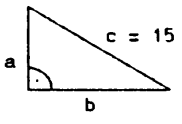
$4\cdot 2 = 3\cdot b$

$3b = 8 \;/:3$

$b = \frac{8}{3}\,cm$

9.

HIPOTENUZA JE NAJDULJA STRANICA PRAVOKUTNOG TROKUTA



zadano

$c = 15$

$\sin\alpha = \frac{2}{3}$

kako je $\sin\alpha = \frac{a}{c}$ i $\sin\alpha = \frac{2}{3}$

tada je $\frac{a}{c} = \frac{2}{3}$ uvrstimo $c = 15$

$\frac{a}{15} = \frac{2}{3} \;/\cdot 15$

$a = \frac{2\cdot 15}{3}$

$a = 10$

dalje po pitagorinom teoremu

$c^2 = a^2 + b^2$ imamo $b^2 = c^2 - a^2$

$b^2 = 15^2 - 10^2$

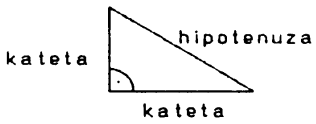
$b^2 = 225 - 100$

$b^2 = 125 \;/\sqrt{}$

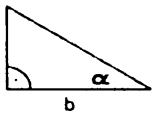
$b = \sqrt{125} = \sqrt{25\cdot 5}$

$b = 5\sqrt{5}$

10.



$b = 15 \rightarrow$ jedna kateta sada je svejedno da li je a ili b



$\rightarrow$ kut prileţeći kateti b je α

$$\sin\alpha = \frac{4}{5}$$

$$\sin\alpha = \frac{\text{kateta nasuprot kuta}}{\text{hipotenuza}} = \frac{a}{c}$$

pa iz toga dobijemo

$$\sin\alpha = \frac{4}{5} \quad i \quad \sin\alpha = \frac{a}{c}$$

$$\frac{a}{c} = \frac{4}{5} \quad / \cdot c$$

$$a = \frac{4}{5}c$$

$$c^2 = a^2 + b^2$$

$$c^2 = \left(\frac{4}{5}c\right)^2 + 15^2$$

$$c^2 = \frac{16}{25}c^2 + 225$$

$$c^2 - \frac{16}{25}c^2 = 225$$

$$\left(1 - \frac{16}{25}\right) \cdot c^2 = 225$$

$$\frac{9}{25}c^2 = 225 \quad / \cdot \frac{25}{9}$$

$$c^2 = \frac{225 \cdot 25}{9} = \frac{9 \cdot 25 \cdot 25}{9}$$

$$c^2 = 625 \quad / \sqrt{\ }$$

$$c = 25 \qquad a = \frac{4}{5}c$$

$$a = \frac{4}{5} \cdot 25 = 20$$

11.

$$\cos\alpha = \frac{3}{4} \qquad b = 9$$

$$\cos\alpha = \frac{b}{c} \quad i \quad \cos\alpha = \frac{3}{4}$$

$$\frac{b}{c} = \frac{3}{4}$$

$$\frac{9}{c} = \frac{3}{4} \quad / \cdot 4 \cdot c$$

$$36 = 3 \cdot c \quad / : 3$$

$$12 = c \rightarrow c = 12 \rightarrow$$

$\rightarrow$

$$c^2 = a^2 + b^2$$

$$a^2 = c^2 - b^2$$

$$a^2 = 12^2 - 9^2$$

$$a^2 = 144 - 81$$

$$a^2 = 63 \quad / \sqrt{\ }$$

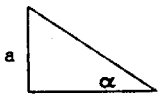
$$a = \sqrt{63} = \sqrt{9 \cdot 7}$$

$$a = 3\sqrt{7}$$

12.

zadano : kateta = 24

cos kuta nasuprotan toj kateti je $\frac{5}{13}$



uzmimo da je zadana kateta = a

a = 24 pa je njoj nasuprotan kut = α

pa je ustvari zadatak zadan ovako:

$a = 24$ i $\cos\alpha = \frac{5}{13}$

iz $\cos\alpha = \frac{b}{c}$ i $\cos\alpha = \frac{5}{13}$

imamo $\frac{b}{c} = \frac{5}{13} \quad / \cdot c$

$b = \frac{5}{13} c$ → → dalje → → →

→→ $c^2 = a^2 + b^2$

$c^2 = 24^2 + \left(\frac{5}{13} c\right)^2$

$c^2 = 576 + \frac{25}{169} c^2$

$c^2 - \frac{25}{169} c^2 = 576$

$\left(1 - \frac{25}{169}\right) \cdot c^2 = 576$

$\frac{144}{169} \cdot c^2 = 576 \quad / \cdot \frac{169}{144}$

$c^2 = 676 \quad / \sqrt{\ }$

$c = 26$

13.

hipotenuza = 25 → c = 25

$\operatorname{tg}\alpha = 3\frac{3}{7} = \frac{3 \cdot 7 + 3}{7} = \frac{24}{7}$

$\operatorname{tg}\alpha = \frac{a}{b}$ i $\operatorname{tg}\alpha = \frac{24}{7}$

$\frac{a}{b} = \frac{24}{7} \quad / \cdot b$

$a = \frac{24}{7} b$

$c^2 = a^2 + b^2$

$25^2 = \left(\frac{24}{7} b\right)^2 + b^2$

$625 = \frac{576}{49} b^2 + b^2$

$625 = \left(\frac{576}{49} + 1\right) \cdot b^2$

$625 = \frac{625}{49} b^2 \quad / \cdot \frac{49}{625}$

$b^2 = 625 \cdot \frac{49}{625}$

$b^2 = 49 \quad / \sqrt{\ }$

$b = 7 \qquad a = \frac{24}{7} b$

$a = \frac{24}{7} \cdot 7$

$a = 24$

14.

$\operatorname{ctg}\alpha = \frac{12}{5} \qquad b = 48$

$\operatorname{ctg}\alpha = \frac{b}{a}$ i $\operatorname{ctg}\alpha = \frac{12}{5}$

$\frac{b}{a} = \frac{12}{5} \quad / \cdot a$

$b = \frac{12}{5} a$

$48 = \frac{12}{5} a \quad / \cdot \frac{5}{12}$

$a = 48 \cdot \frac{5}{12} = \frac{12 \cdot 4 \cdot 5}{12}$

$a = 20$

$c^2 = a^2 + b^2$

$c^2 = 20^2 + 48^2$

$c^2 = 400 + 2304$

$c^2 = 2704 \quad / \sqrt{\ }$

$c = 52$

15.

Kada je zadani kut šiljat to znači da je α u intervalo od 0° do 90° .

vrijednosti sa $\sin\alpha$ i $\cos\alpha$ u tom intervalu su od 0 do 1 pa pišemo : $0 < \sin\alpha < 1$ i $0 < \cos\alpha < 1$

1.) $\sin\alpha = \frac{1}{1-a}$ $\quad 0 < \sin\alpha < 1$

$0 < \frac{1}{1-a} < 1$

I

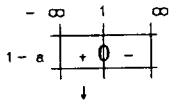
$0 < \frac{1}{1-a}$

$\frac{1}{1-a} > 0$

$1-a = 0$

$-a = -1 \ / \cdot(-1)$

$a = 1$



$a \in \langle -\infty, 1 \rangle$

II

$\frac{1}{1-a} < 1$

$\frac{1}{1-a} - 1 < 0$

$\frac{1-1\cdot(1-a)}{1-a} < 0$

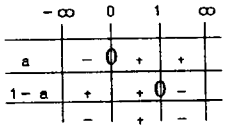
$\frac{1-1+a}{1-a} < 0$

$\frac{a}{1-a} < 0$

$a = 0 \quad 1-a = 0$

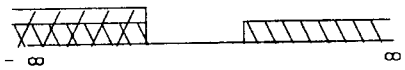
$-a = -1 \ / \cdot(-1)$

$a = 1$



$a \in \langle -\infty, 0 \rangle \cup \langle 1, \infty \rangle$

ukupno rješenje je presjek I i II



$a \in \langle -\infty, 0 \rangle$

rješenje možemo pisati i $a < 1$

2.) $\cos\alpha = \frac{2a}{a^2+1}$ $\quad 0 < \cos\alpha < 1$

$0 < \frac{2a}{a^2+1} < 1$

I

$0 < \frac{2a}{a^2+1}$

II

$\frac{2a}{a^2+1} < 1$

kako je nazivnik a^2+1 pozitivan za bilo koji $a \in R$ to obadvije nejednadžbe možemo množiti sa a^2+1

$0 < \frac{2a}{a^2+1} \ / \cdot(a^2+1)$ $\quad \frac{2a}{a^2+1} < 1 \ / \cdot(a^2+1)$

$0 < 2a$

$2a > 0 \ / : 2$

$a > 0$

$2a < a^2 + 1$

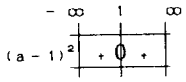
$0 < a^2 - 2a + 1$

$a^2 - 2a + 1 > 0$

$(a-1)^2 > 0$

nul točka $a - 1 = 0$

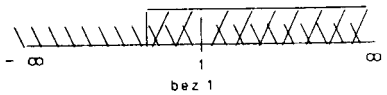
$a = 1$



$a > 0$

$a \in R \setminus \{1\}$

ukupno rješenje je presjek I i II



bez 1

$a \in \langle 0, \infty \rangle \setminus \{1\}$

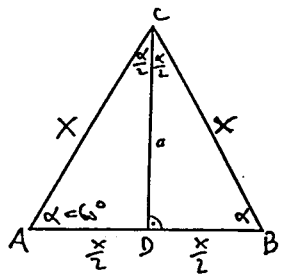
ili

$a > 0 \ i \ a \neq 0$

4.2. Vrijednosti trigonometrijskih funkcija kuteva od $30^0, 45^0, 60^0$

1.

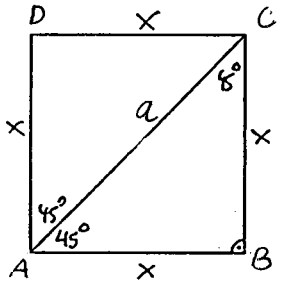
1.)



STRANICU OZNAČIMO SA (x) TAKO DA NE RJEŠAVAMO...
IZ PRAVOKUTNOG TROKUTA ADC IMAMO:

$$\sin\alpha = \frac{a}{x} \quad / \cdot x$$
$$x \cdot \sin\alpha = a \quad / : \sin\alpha$$
$$x = \frac{a}{\sin\alpha}$$
$$x = \frac{a}{\sin 60^\circ} = \frac{a}{\frac{\sqrt{3}}{2}} = \frac{\frac{a}{1}}{\frac{\sqrt{3}}{2}} = \frac{2a}{\sqrt{3}}$$
$$x = \frac{2a}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2a\sqrt{3}}{\sqrt{3}^2} = \frac{2\sqrt{3}}{3}a$$
$$x = \frac{2\sqrt{3}}{3}a$$

2.)

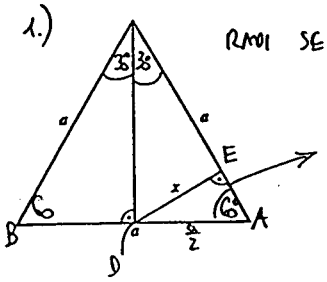


STRANICU OZNAČIMO SA X
IZ PRAVOKUTNOG TROKUTA ABC IMAMO
PO PITAGORINOM TEOREMU

$$a^2 = x^2 + x^2$$
$$a^2 = 2x^2 \quad / : 2$$
$$\frac{a^2}{2} = x^2 \Rightarrow \quad x^2 = \frac{a^2}{2} \quad / \sqrt{\ }$$
$$x = \frac{\sqrt{a^2}}{\sqrt{2}} = \frac{a}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$
$$x = \frac{a\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \cdot a$$

2.

1.)



RIJEČ JE O JEDNAKOSTRANIČNOM TROKUTU ČIJI JE $\alpha = 60^\circ$

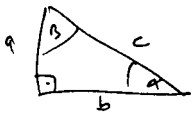
IZ PRAVOKUTNOG TROKUTA AED IMAMO

$$\sin 60^\circ = \frac{x}{\frac{a}{2}} \longrightarrow \quad \frac{x}{\frac{a}{2}} = \frac{\frac{x}{1}}{\frac{a}{2}} = \frac{2x}{a}$$
$$\sin 60^\circ = \frac{2x}{a}$$
$$\frac{\sqrt{3}}{2} = \frac{2x}{a} \quad / \cdot \frac{a}{2}$$
$$\frac{\sqrt{3}}{2} \cdot \frac{a}{2} = x$$
$$x = \frac{a\sqrt{3}}{4} = \frac{\sqrt{3}}{4}a$$

4.3. Računanje vrijednosti trigonometrijskih funkcija

4.3.

6. **1)** $a = 5$, $c = 13$



① $b^2 = c^2 - a^2$

$b^2 = 13^2 - 5^2$

$b^2 = 144 \ /\sqrt{}$

$b = 12$

② $\sin\alpha = \frac{a}{c}$

$\sin\alpha = \frac{5}{13}$

$\sin\alpha = 0,38461538 \ /\sin^{-1}$

$\alpha = 22°37'12''$ → OVO RAČUNAMO RAČUNALOM (DIGITRON)

③ $\cos\alpha = \frac{b}{c}$

$\cos\alpha = \frac{12}{13}$

$\cos\alpha = 0,9230769 \ /\cos^{-1}$

$\alpha = 22°37'12''$

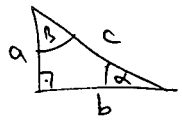
③ $\sin\beta = \frac{b}{c}$

$\sin\beta = \frac{12}{13}$

$\sin\beta = 0,9230769 \ /\sin^{-1}$

$\beta = 67°22'48''$

2) $b = 8$ $c = 15$



① $\sin\alpha = \frac{a}{c}$

$\sin\alpha = \frac{\sqrt{161}}{15}$

$\sin\alpha = 0,845905 \ /\sin^{-1}$

$\alpha = 57°46'9''$

② $\sin\beta = \frac{b}{c}$

$\sin\beta = \frac{8}{15}$

$\sin\beta = 0,53333 \ /\sin^{-1}$

$\beta = 32°13'51''$

$a^2 = c^2 - b^2$

$a^2 = 15^2 - 8^2 = 225 - 64$

$a^2 = 161 \ /\sqrt{}$

$a = \sqrt{161}$

3.) $a = 20$

$b = 21$

$\operatorname{tg}\alpha = \frac{a}{b}$

$\operatorname{tg}\alpha = \frac{20}{21}$

$\operatorname{tg}\alpha = 0,9523809 \ /\operatorname{tg}^{-1}$

$\alpha = 43°36'10''$

U PRAVOKUTNOM TROKUTU VRIJEDI: $\alpha + \beta = 90°$

PA JE $\beta = 90° - \alpha$

$\beta = 89°59'60' - 43°36'10''$

$\beta = 46°23'50''$

ILI ① $c^2 = a^2 + b^2$

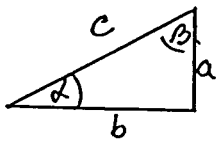
① $\cos\beta = \frac{a}{c}$ ILI $\sin\beta = \frac{b}{c}$

$\beta = 46°23'50''$

* U OVAKVOM TIPU ZADATAKA IMA VIŠE NAČINA RJEŠAVANJA DO TOČNOG RJEŠENJA U SVAKOM ZADATKU 1, 2, 3 PRIKAZAO SAM JEDAN OD NAVEDENIH NAČINA SVI NAČINI RJEŠAVANJA SU DOBRI ALI ONAJ TOČNO RJEŠENJE ....

7. zadatak

Pravokutan trokut



$\alpha + \beta = 90°$

$\beta = 90° - \alpha$

1) $a = 3cm$
$b = 4cm$

$c = ?$
$\alpha = ?$
$\beta = ?$

$tg\,\alpha = \frac{a}{b}$

$tg\,\alpha = \frac{3}{4}$

$tg\,\alpha = 0{,}75$

$\alpha = tg^{-1} 0{,}75$

$\alpha = 36{,}8699°$

$\alpha = 36°52'12''$

$\beta = 90° - \alpha$

$\beta = 90° - 36°52'12''$

$\beta = 53{,}1301°$

$\beta = 53°7'48''$

$\sin\alpha = \frac{a}{c} \;/\cdot c$

$\sin\alpha \cdot c = a$

$c = \frac{a}{\sin\alpha} = \frac{3}{\sin 36°52'12''}$

$c = \frac{3}{0{,}6} = 5\,cm$

2) $a = 5cm$
$c = 7cm$

$b = ?$
$\alpha = ?$
$\beta = ?$

$\sin\alpha = \frac{a}{c}$

$\sin\alpha = \frac{5}{7} = 0{,}71429$

$\alpha = \sin^{-1} 0{,}71429$

$\alpha = 45{,}5847°$

$\alpha = 45°35'5''$

$tg\,\alpha = \frac{a}{b}$

$b = \frac{a}{tg\,\alpha} = \frac{5}{tg\,45°35'5''}$

$b = \frac{5}{1{,}0206}$

$b = 4{,}899\,cm$

$\beta = 90° - \alpha$

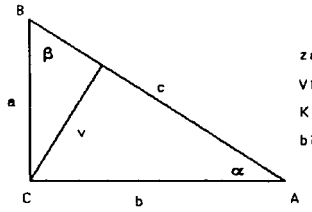
$\beta = 90° - 45°35'5''$

$\beta = 44{,}4153°$

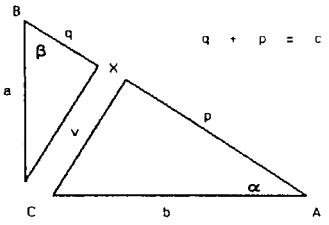
$\beta = 44°24'55''$

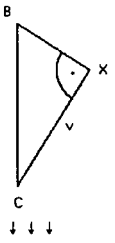
18. zadano $v = 20,4$ cm i $\alpha = 32^\circ 24'$

1.)

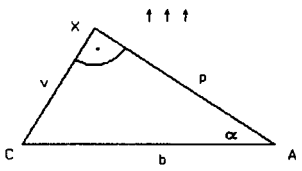


zadana je visina i jedan šiljasti kut . . .
Visina je okomica iz vrha C na stranicu c
Kada bi prerezali taj trokut po visini dobili
bismo dva pravokutna trokuta → → →





uzmimo sada prvi trokut AXC → to je pravokutan trokut sa hipotenuzom b



$$\operatorname{tg}\alpha = \frac{v}{p} \quad / \cdot p$$

$$p \cdot \operatorname{tg}\alpha = v \quad / : \operatorname{tg}\alpha$$

$$p = \frac{v}{\operatorname{tg}\alpha}$$

$$p = \frac{20,4}{\operatorname{tg} 32^\circ 40'} = \frac{20,4}{0,64116734} = 31,8169667$$

$p = 31,82$ cm

nakon toga uzmemo drugi trokut CXB

$\alpha + \beta = 90^\circ$

$\beta = 90^\circ - \alpha$

$\beta = 89^\circ 60' - 32^\circ 40'$

$\beta = 57^\circ 20'$

$$\operatorname{tg}\beta = \frac{v}{q} \quad / \cdot q$$

$$q \cdot \operatorname{tg}\beta = v \quad / : \operatorname{tg}\beta$$

$$q = \frac{v}{\operatorname{tg}\beta}$$

$$q = \frac{20,4}{\operatorname{tg} 57^\circ 20'} = \frac{20,4}{1,5596552}$$

$q = 13,079814$

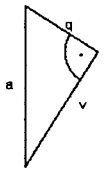
$q = 13,08$ cm

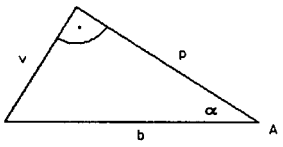
$p = 31,82$ cm $\qquad q = 13,08$ cm

$c = p + q$

$c = 31,82 + 13,08$

$c = 44,9$ cm





$a^2 = v^2 + q^2$

$a^2 = 20,4^2 + 13,08^2$

$a^2 = 416,15 + 171,0864$

$a^2 = 587,2464 \quad / \sqrt{\ }$

$a = 24,23316735$

$a = 24,23$ cm

$b^2 = v^2 + p^2$

$b^2 = 20,4^2 + 31,82^2$

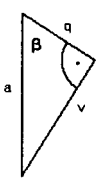
$b^2 = 416,16 + 1012,5124$

$b^2 = 1428,6724 \quad / \sqrt{\ }$

$b = 37,797783$

$b = 37,80$ cm

OVAJ zadatak mogli smo riješiti i na drugi način tako da prvo računamo a i b stranice pa tek onda c stranu . . .



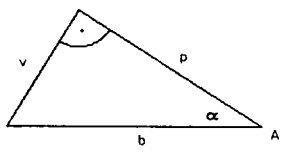
$\alpha + \beta = 90$

$\beta = 57^\circ 20'$

$\sin\beta = \frac{v}{a}$

$$a = \frac{v}{\sin\beta} = \frac{20,4}{\sin 57^\circ 20'} = \frac{20,4}{0,84182494}$$

$a = 24,23$ cm



$$\operatorname{tg}\alpha = \frac{v}{p} \quad / \cdot p$$

$$p = \frac{v}{\operatorname{tg}\alpha}$$

isto kao i gore . . .

pa sada računamo c stranicu preko $c^2 = a^2 + b^2$ bez računanja p i q . . . obadva načina su dc
ovaj drugi je nešto kraći . . .

2.) zadano $v = 5,26$ cm i $\beta = 65^\circ 30'$ → tehnika je ista kao u 1.) prvo izračunamo α

$\alpha = 90^\circ - \beta$

$\alpha = 89^\circ 60' - 65^\circ 30'$

$\alpha = 24^\circ 30'$

$q = \frac{v}{\operatorname{tg}\beta}$

$q = \frac{5,26}{\operatorname{tg} 65^\circ 30'} = \frac{5,26}{2,19429973}$

$q = 2,397120$

$q = 2,40$ cm

$p = \frac{v}{\operatorname{tg}\alpha}$

$p = \frac{5,26}{\operatorname{tg} 24^\circ 30'} = \frac{5,26}{0,455726255}$

$p = 11,542$ cm

$c = p + q$

$c = 11,542 + 2,40$

$c = 13,942$

$c = 13,94$ cm

$a^2 = v^2 + q^2$

$a^2 = 5,26^2 + 2,4^2$

$a^2 = 27,6676 + 5,76$

$a^2 = 33,4276 \quad /\sqrt{\ }$

$a = 5,78$ cm

$b^2 = c^2 - a^2$

$b^2 = 13,94^2 - 5,78^2$

$b^2 = 194,3236 - 33,4084$

$b^2 = 160,9152 \quad /\sqrt{\ }$

$b = 12,69$ cm

19. pravokutan trokut

$P = 22\ cm^2$

$\alpha = 56^\circ 40'$

$a = ?$

$b = ?$

$c = ?$

$P = \frac{a \cdot b}{2} \quad /\cdot 2$

$2P = a \cdot b$

$\sin\alpha = \frac{a}{c}$

$a = c \cdot \sin\alpha$

$a = c \cdot \sin 56^\circ 40'$

$a = c \cdot 0,83549$

$a = 0,83549\ c$

$\cos\alpha = \frac{b}{c}$

$b = c \cdot \cos\alpha$

$b = c \cdot \cos 56^\circ 40'$

$b = 0,5495 \cdot c$

$2P = a \cdot b$

$2 \cdot 22 = 0,83549 \cdot c \cdot 0,5495 \cdot c$

$44 = 0,4591\ c^2 \quad /: 0,4591$

$c^2 = \frac{44}{0,4591}$

$c^2 = 95,839686 \quad /\sqrt{\ }$

$c = 9,78977$ cm

$c = 9,8$ cm

$b = 0,5495 \cdot c$

$b = 0,5495 \cdot 9,8$ cm

$b = 5,3851$ cm

$b = 5,38$ cm

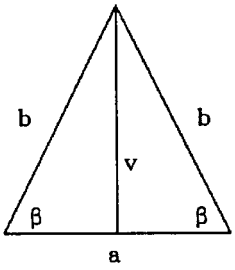
$a = 0,83549 \cdot c$

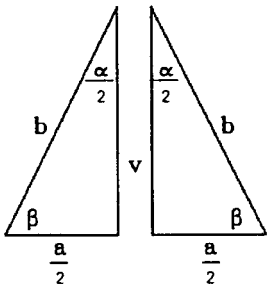
$a = 0,83549 \cdot 9,8$ cm

$a = 8,18$ cm

4.5. Primjene u planimetriji

1.





RASTAVIMO OVAJ JEDNAKOKRAČAN TROKUT PO VISINI NA DVA PRAVOKUTNA TROKUTA

$\sin\beta = \frac{v}{b}$ $\quad \cos\beta = \frac{a}{2\cdot b}$

$\operatorname{tg}\beta = \frac{2\cdot v}{a}$ $\quad \operatorname{ctg}\beta = \frac{a}{2\cdot v}$

a) $a = 6{,}5$ cm

$b = 11$ cm

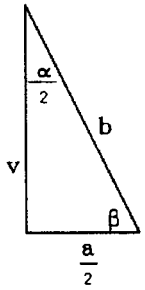
$\cos\beta = \frac{a}{2\cdot b}$

$\cos\beta = \frac{6{,}5}{2\cdot 11} = \frac{6{,}5}{22}$

$\cos\beta = 0{,}295445 \quad / \cos^{-1}$

$\beta = 72^\circ 48' 55''$

$\beta = 72^\circ 49'$



$\frac{\alpha}{2} = 90^\circ - 72^\circ 49'$

$\frac{\alpha}{2} = 89^\circ 60' - 72^\circ 49'$

$\frac{\alpha}{2} = 17^\circ 11' \quad / \cdot 2 \quad \rightarrow \quad \alpha = 34^\circ 22'$

b) $a = 22{,}7$ cm

$b = 15{,}2$ cm

$\cos\beta = \frac{a}{2\cdot b}$

$\cos\beta = \frac{22{,}7}{2\cdot 15{,}2} = \frac{22{,}7}{30{,}4}$

$\cos\beta = 0{,}74671052 \quad / \cos^{-1}$

$\beta = 41^\circ 41' 38''$

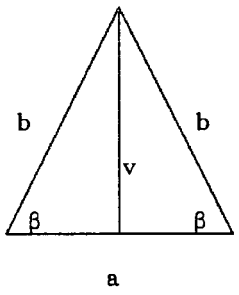
$\beta = 41^\circ 42'$

$\frac{\alpha}{2} = 89^\circ 60' - 41^\circ 42'$

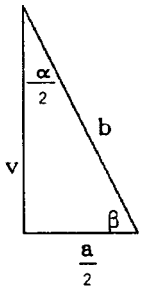
$\frac{\alpha}{2} = 48^\circ 18' \quad / \cdot 2$

$\alpha = 96^\circ 36'$

2.



← iz trokuta izvadimo trokut →



$\sin\frac{\alpha}{2} = \frac{\frac{a}{2}}{b} = \frac{a}{2\cdot b}$

$\sin\frac{\alpha}{2} = \frac{a}{2\cdot b}$

$\cos\frac{\alpha}{2} = \frac{v}{b}$

1) $\alpha = 140^\circ \quad \rightarrow \quad \frac{\alpha}{2} = 70^\circ$

$a = 20$ cm

$\sin\frac{\alpha}{2} = \frac{a}{2\cdot b} \quad / \cdot b$

$b \cdot \sin\frac{\alpha}{2} = \frac{a}{2} \quad / : \sin\frac{\alpha}{2}$

$b = \frac{a}{2\cdot \sin\frac{\alpha}{2}}$

$b = \frac{20}{2\cdot \sin 70^\circ} = \frac{20}{2\cdot 0{,}939693} = 10{,}64178$

$b = 10{,}64$ cm

$\beta = 90^\circ - \frac{\alpha}{2} = 90^\circ - 70^\circ$

$\beta = 20^\circ$

2) $\alpha = 55^\circ \quad \rightarrow \quad \frac{\alpha}{2} = 27^\circ 30'$

$a = 8{,}5$ cm

$b = \frac{a}{2\cdot \sin\frac{\alpha}{2}}$

$b = \frac{8{,}5}{2\cdot \sin 27^\circ 30'} = \frac{8{,}5}{2\cdot 0{,}4617486}$

$b = \frac{8{,}5}{0{,}923497} = 9{,}2041$

$b = 9{,}2$ cm

$\beta = 90^\circ - \frac{\alpha}{2}$

$\beta = 89^\circ 60 - 27^\circ 30$

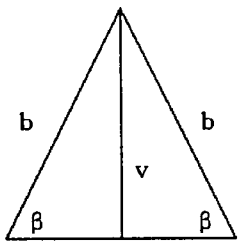
$\beta = 62^\circ 30'$

3.

1)

$b = 45$ cm

$\beta = 12^\circ$



$\cos\beta = \frac{a}{2 \cdot b} \quad / \cdot 2 \cdot b$

$2 \cdot b \cdot \cos\beta = a$

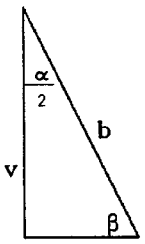
$a = 2 \cdot b \cdot \cos\beta$

$a = 2 \cdot 45 \cdot \cos 12^\circ$

$a = 90 \cdot 0{,}9781476$

$a = 88{,}03328$

$a = 88{,}03$ cm



$\alpha + 2 \cdot \beta = 180^\circ$

$\alpha = 180^\circ - 2 \cdot \beta$

$\alpha = 180^\circ - 2 \cdot 12^\circ = 180^\circ - 24^\circ$

$\alpha = 156^\circ$

2) $b = 5{,}2$ cm

$\beta = 67^\circ 20'$

$a = 2 \cdot b \cdot \cos\beta$

$a = 2 \cdot 5{,}2 \cdot \cos 67^\circ 20'$

$a = 10{,}4 \cdot 0{,}3853693$

$a = 4{,}00784$

$a = 4{,}01$ cm

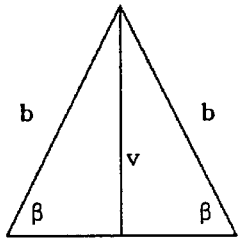
$\alpha = 180^\circ - 2 \cdot \beta$

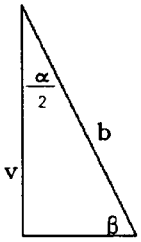
$\alpha = 180^\circ - 2 \cdot 67^\circ 20' = 179^\circ 60' - 134^\circ 40'$

$\alpha = 45^\circ 20'$

4.

1)





zaano α i v : računamo po ↑ →

$\alpha = 101^\circ \rightarrow \frac{\alpha}{2} = 50^\circ 30'$

$v = 15$ cm

$a = 2 \cdot v \cdot \operatorname{tg}\frac{\alpha}{2}$

$a = 2 \cdot 15 \cdot \operatorname{tg} 50^\circ 30'$

$a = 30 \cdot 1{,}213097$

$a = 36{,}3929$

$a = 36{,}39$ cm

$\operatorname{tg}\frac{\alpha}{2} = \frac{\frac{a}{2}}{v}$

$\operatorname{tg}\frac{\alpha}{2} = \frac{a}{2 \cdot v} \quad / \cdot 2 \cdot v$

$2 \cdot v \cdot \operatorname{tg}\frac{\alpha}{2} = a$

$a = 2 \cdot v \cdot \operatorname{tg}\frac{\alpha}{2}$

$\sin\frac{\alpha}{2} = \frac{a}{2 \cdot b}$

$\cos\frac{\alpha}{2} = \frac{v}{b} \quad / \cdot b \rightarrow b \cdot \cos\frac{\alpha}{2} = v \quad / : \cos\frac{\alpha}{2}$

$b = \frac{v}{\cos\frac{\alpha}{2}}$

$b = \frac{15}{\cos 50^\circ 30'} = \frac{15}{0{,}636078} = 23{,}582$

$b = 23{,}58$ cm

2) $\alpha = 33^\circ \rightarrow \frac{\alpha}{2} = 16^\circ 30'$

$v = 112$ cm

$a = 2 \cdot v \cdot \operatorname{tg}\frac{\alpha}{2}$

$a = 2 \cdot 112 \cdot \operatorname{tg} 16^\circ 30'$

$a = 224 \cdot 02962135 = 66{,}35182$

$a = 66{,}35$ cm

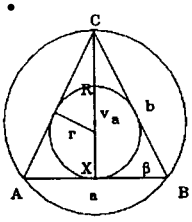
$b = \frac{v}{\cos\frac{\alpha}{2}}$

$b = \frac{112}{\cos 16^\circ 30'} = \frac{112}{0{,}9588197} = 116{,}810278$

$b = 116{,}81$ cm

18.

- zadano: $v_a = 8$ cm i osnovica : krak = 6 : 5



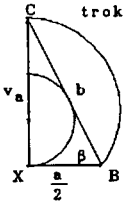
1. $a : b = 6 : 5$

$\frac{a}{b} = \frac{6}{5} \quad / \cdot b$

$a = \frac{6}{5} b$

PA JE $\frac{a}{2} = \frac{3b}{5}$

2. - iz jednakokračnog trokuta izdvojimi pravokutni trokut XBC i pomoću njega dodemo do kuta β . . .



$\cos\beta = \frac{\frac{a}{2}}{b}$

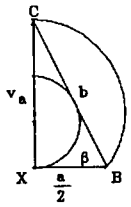
$\cos\beta = \frac{\frac{3b}{5}}{b} = \frac{3b}{5b}$

$\cos\beta = \frac{3}{5}$

$\cos\beta = 0{,}6 \quad / \cos^{-1}$

$\beta = 53°7'48''$

3.



$\sin\beta = \frac{v}{b} \quad / \cdot \frac{b}{\sin\beta}$

$b = \frac{v}{\sin\beta}$

$b = \frac{8}{\sin 53°7'48''} = \frac{8}{0{,}79999893} = 10$

$b = 10$ cm

4. I SADA IZ ZADATKA 5.82. IMAMO FORMULU :

$R = \frac{b}{2 \cdot \sin\beta}$

$R = \frac{10}{2 \cdot \sin 53°7'48''} = \frac{5}{0{,}79999893} = 6{,}250836$

$R = 6{,}25$ cm

5. $a = \frac{6}{5} b = \frac{6}{5} \cdot 10 = 6 \cdot 2$

$a = 12$ cm

6. I SADA IZ ZADATKA 5.82. IMAMO FORMULU :

$r = \frac{a \cdot \operatorname{tg}\frac{\beta}{2}}{2}$

$r = \frac{12 \cdot \operatorname{tg} 26°33'54''}{2} = 6 \cdot 0{,}4999998883$

$r = 2{,}99999330$

$r = 3$ cm

19.

- zadano $O = 2$ dm , $P = \frac{6}{25}$ dm²

$O = 2 \cdot (a + b)$

$2 = 2 \cdot (a + b) \quad / : 2$

$1 = a + b$

$1 - a = b \rightarrow \rightarrow \rightarrow \rightarrow$

$P = a \cdot b$

$\frac{6}{25} = a \cdot b$

$\frac{6}{25} = a \cdot (1 - a) \quad / \cdot 25$

$6 = 25a \cdot (1 - a)$

$6 = 25a - 25a^2$

$25a^2 - 25a + 6 = 0$

$a_{1,2} = \frac{-(-25) \pm \sqrt{(-25)^2 - 4 \cdot 25 \cdot 6}}{2 \cdot 25} = \frac{25 \pm \sqrt{25}}{50} = \frac{25 \pm 5}{50}$

$a_1 = \frac{25 + 5}{50} = \frac{30}{50} = \frac{3}{5}$ $\quad a_2 = \frac{25 - 5}{50} = \frac{20}{50} = \frac{2}{5}$

$a_1 = \frac{3}{5}$ $\quad a_2 = \frac{2}{5}$

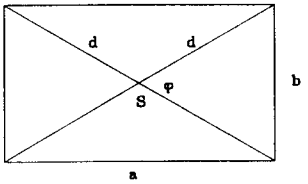
$b = 1 - a$

$b_1 = 1 - \frac{3}{5} = \frac{5 - 3}{5} = \frac{2}{5}$ $\quad b_2 = 1 - \frac{2}{5} = \frac{3}{5}$

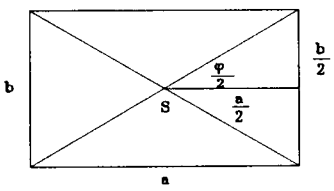
$b_1 = \frac{2}{5}$ $\quad b_2 = \frac{3}{5}$ $\rightarrow\rightarrow\rightarrow$ uzmimo samo a_1 i b_1 jer za a_2 i b_2 radi se o istom pravokutniku . .

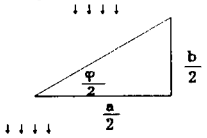
- $a = \frac{3}{5}$ i $b = \frac{2}{5}$

- nacrtajmo taj pravokutnik označimo stranice i kut . . .



- iz sjecišta dijagonala spustimo visinu na b . . . dobili smo pravokutni trokut pa kut φ dobijemo:





$\operatorname{tg}\frac{\varphi}{2} = \frac{b}{a}$, $\operatorname{tg}\frac{\varphi}{2} = \frac{\frac{2}{5}}{\frac{3}{5}} = \frac{2}{3}$, $\operatorname{tg}\frac{\varphi}{2} = 0{,}66666 \quad / \operatorname{tg}^{-1}$, $\frac{\varphi}{2} = 33°41'24'' \quad / \cdot 2$

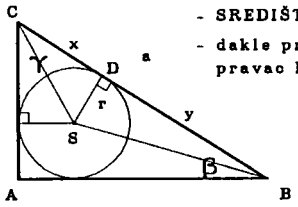
$\varphi = 67°22'48''$

4.5.

ovo je samo jedna manji dio detaljno riješenih zadataka …kommpletno sva rješenja mogu se nabaviti u štampanoj varijanti
- za detalje kontaktirajte nas na telefon – 01-4578-431 ili 098-237-534

36. zadano : $\beta = 50°$, $\gamma = 74°$, polumjer upisane kružnice $r = 25$ cm , $a = ?$

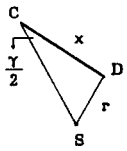
naocrtajmo sliku :



- SREDIŠTE TROKUTU UPISANE KRUŽNICE JE SJECIŠTE SIMETRALA KUTEVA -
- dakle pravac kroz točke C i S dijeli kut γ na dva jednaka djela , isto tako pravac kroz točke B i S dijeli kut β na dva jednaka djela . . .

- iz Δ ABC izdvojimo dva pravokutna Δ

- Δ CSD



$\operatorname{tg} \frac{\gamma}{2} = \frac{r}{x}$

$\operatorname{tg} \frac{74°}{2} = \frac{25}{x} \quad / \cdot x$

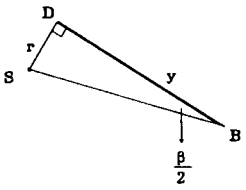
$x \cdot \operatorname{tg} 37° = 25$

$x \cdot 0{,}75355 = 25 \quad / : 0{,}75355$

$x = 33{,}17629885$

$x = 33{,}18$ cm

- Δ BSD



$\operatorname{tg} \frac{\beta}{2} = \frac{r}{y}$

$\operatorname{tg} \frac{50°}{2} = \frac{25}{y} \quad / \cdot y$

$y \cdot \operatorname{tg} 25° = 25$

$y \cdot 0{,}46631 = 25 \quad / : 0{,}46631$

$y = 53{,}6124$

$y = 53{,}61$ cm

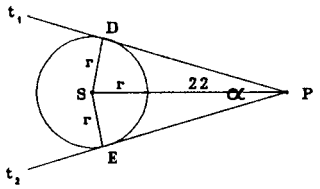
$c = x + y$

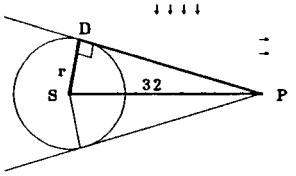
$c = 33{,}18 + 53{,}61$

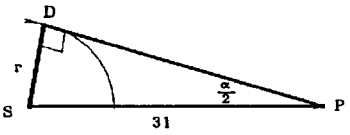
$c = 86{,}79$ cm

U RJEŠENJIMA ZBIRKE FORMULA JE OK. ALI UMJESTO α TREBA PISATI β I RAČUN IM NIJE DOBAR PROVJERITE SAMI UZMITE LOG. TABLICE I OČITAJTE ctg 37° i ctg 25° I UVRSTITE IH U NJIHOVU FORMULU I DOBITI ĆETE MOJ c . . .

37. naocrtajmo sliku : prvo naocrtamo kružnicu zatim polumjer produžimo za 22 cm jer je udaljenost od središta S do točke P 31 cm kako je r = 9 to je udaljenost točke P od kružnice = 22 cm , dalje iz točke P povučemo tangente t na kružnicu , točke dodira su D i E , u tim točkama r je okomit na tangente . . . uočimo pravokutni Δ PSD







- zašto je r okomit na tangentu u točci D ?
- jer je pravac kroz točke S i D (normala) okomit na tangentu a polumjer r nalazi se na tom pravou . . .

imamo : $\sin \frac{\alpha}{2} = \frac{r}{31}$

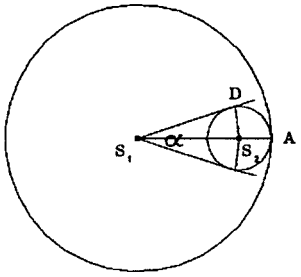
$\sin \frac{\alpha}{2} = \frac{9}{31}$

$\sin \frac{\alpha}{2} = 0{,}29032 \quad / \sin^{-1}$

$\frac{\alpha}{2} = 16°52'36'' \quad / \cdot 2$

$\alpha = 33°45'12''$

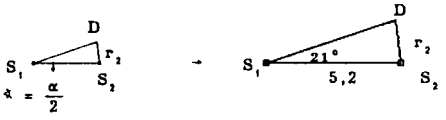
38. naocrtajmo sliku : - kružnice se diraju iznutra $d(S_1S_2) = r_1 - r_2$



$r_1 - r_2 = 5{,}2$ cm

$\alpha = 42°$

- treba uočiti pravokutni Δ S_1S_2D
- izdvojimo ga



1. $\operatorname{tg} \frac{\alpha}{2} = \frac{r_2}{r_1 - r_2} \rightarrow \quad \sin 21° = \frac{r_2}{5{,}2} \quad / \cdot 5{,}2$

$5{,}2 \cdot \sin 21° = r_2$

$r_2 = 5{,}2 \cdot 0{,}35837 = 1{,}863524$

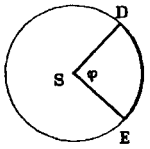
$r_2 = 1{,}86$ cm

2. $r_1 - r_2 = 5{,}2$ cm

$r_1 - 1{,}86 = 5{,}2$

$r_1 = 5{,}2 + 1{,}86$

$r_1 = 7{,}06$ cm



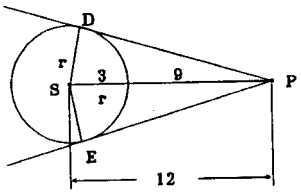
$l(\varphi)$ → duljina kružnog luka $\qquad l(\varphi) = \frac{r \cdot \pi}{180^\circ} \cdot \varphi$

- duljina kružnog luka je udaljenost točke D do E kada se ide po kružnici normalno sa onog dijela "polukruga" kojeg određuje kut φ . . .

39.

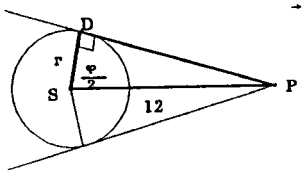
• zadano $r = 3$ cm , $d(SP) = 12$ cm , traži se duljina kružnog luka . . .

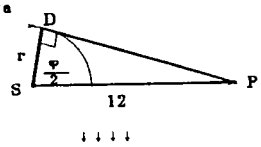
- slika:



sl.1.

- na sl.1. uoči pravokutni Δ DSP → izdvojimo ga





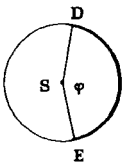
$$\cos\frac{\varphi}{2} = \frac{r}{12}$$

$$\cos\frac{\varphi}{2} = \frac{3}{12}$$

$$\cos\frac{\varphi}{2} = 0{,}25 \quad / \cos^{-1}$$

$$\frac{\varphi}{2} = 75^\circ 31' 21'' \quad / \cdot 2$$

$$\varphi = 151^\circ 2' 42''$$



$$l(\varphi) = \frac{r \cdot \pi}{180^\circ} \cdot \varphi$$

$$l = \frac{3 \cdot 3{,}14}{180^\circ} \cdot 151^\circ 2' 42''$$

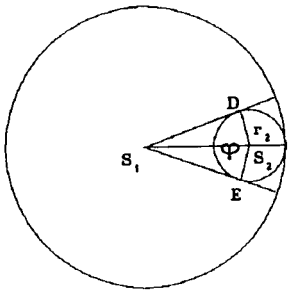
$$l = \frac{9{,}42}{180^\circ} \cdot 151{,}045^\circ = 7{,}904688$$

$$l = 7{,}9 \text{ cm}$$

40.

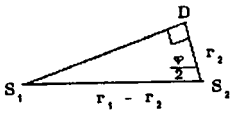
$r_2 = 3$ cm , $r_1 = 7$ cm

- slika :



$d(S_1S_2) = r_1 - r_2$

→ izdvojimo Δ S_1S_2D



$$\cos\frac{\varphi}{2} = \frac{r_2}{r_1 - r_2}$$

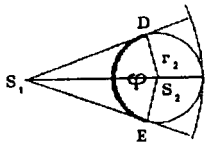
$$\cos\frac{\varphi}{2} = \frac{3}{7-3} = \frac{3}{4}$$

$$\cos\frac{\varphi}{2} = 0{,}75 \quad / \cos^{-1}$$

$$\frac{\varphi}{2} = 41^\circ 24' 35'' \quad / \cdot 2$$

$$\varphi = 82^\circ 49' 10''$$

- duljina luka manje kružnice koja se vidi iz S_1 je :



$$l(\varphi) = \frac{r \cdot \pi}{180^\circ} \cdot \varphi$$

$$l = \frac{3 \cdot 3{,}14}{180^\circ} \cdot 82^\circ 49' 10'' = \frac{9{,}42}{180^\circ} \cdot 82{,}8194444^\circ = 4{,}33421759$$

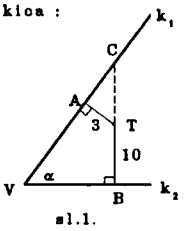
$$l = 4{,}33 \text{ cm}$$

4.5.

61. OVDJE SE POJAVLJUJE JEDAN PROBLEM RJEŠENJA ZBIRKE RADE KAO DA JE ZADANA UDALJENOST OD JEDNOG KRAKA 10 I DRUGOG 2 CM , DOK U ZBIRCI KOJU JA IMAM PIŠE DA JE UDALJENOST OD KRAKA 10 I 3 CM , JA ĆU RADITI PO ZADANOM ZADATKU . . .

- zadano : točka T je udaljena od krakova kuta $\alpha = 75^\circ$ za 3 cm i 10 cm . . .
- traži se udalenost od vrha V kuta α do točke T . . .

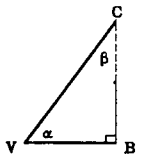
- skica :



sl.1.

- najbliža udaljenost točke od kraka je njena okomita projekcija krak (u točke A i B)
- vrh kuta α označimo sa V
- produžimo pravac kroz točke B i T do kraka k_1 sjecište označimo sa C

1. - iz pravokutnog Δ VBC dodemo do β



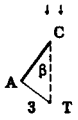
sl.2.

$\beta = 90^\circ - \alpha$

$\beta = 90^\circ - 75^\circ$

$\beta = 15^\circ$

2. - iz sl.1. izdvojimo pravokutni Δ CAT



$\sin\beta = \frac{3}{|CT|} \quad / \cdot \frac{|CT|}{\sin\beta}$

$|CT| = \frac{3}{\sin\beta}$

$|CT| = \frac{3}{\sin 15^\circ} = \frac{3}{0{,}25882}$

$|CT| = 11{,}59 \text{ cm}$

3. - iz sl.2. - imamo :

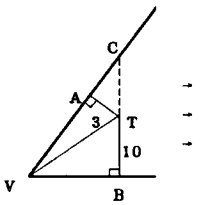
$\operatorname{tg}\beta = \frac{|VB|}{|BC|} \quad / \cdot |VB|$

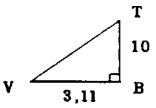
$|VB| = |BC| \cdot \operatorname{tg}\beta$

$|VB| = 11{,}59 \cdot \operatorname{tg} 15^\circ = 11{,}59 \cdot 0{,}26795 = 3{,}1055405$

$|VB| = 3{,}11 \text{ cm}$

4. - i sada tražimo udaljenost vrha V od točke T
- spojimo točke V i T i iz slike izdvijimo pravokutni Δ VBT





- po Pitagorinom teoremu imamo

$|VT|^2 = |VB|^2 + |BT|^2$

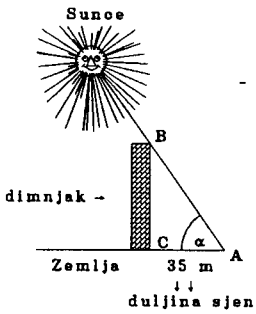
$|VT|^2 = 3{,}11^2 + 10^2 = 9{,}6721 + 100$

$|VT|^2 = 109{,}6721 \quad /\sqrt{\ }$

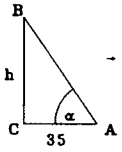
$|VT| = 10{,}4724$

62. - zadano : duljina sjene = 35 m , i kut pod kojim sunčeve zrake padaju na zemlju $\alpha = 55^\circ$, traži se visina dimnjaka . . .

- slika :



- iz ove moje prekrasne slike izdvojimo pravokutni Δ ABC



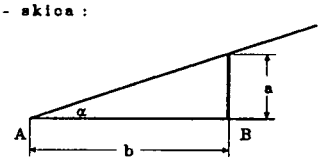
$\operatorname{tg}\alpha = \frac{h}{35} \quad / \cdot 35$

$h = 35 \cdot \operatorname{tg}\alpha = 35 \cdot \operatorname{tg} 55^\circ = 35 \cdot 1{,}428148 = 49{,}98518$

$h = 50 \text{ m}$ → tražena visina dimnjaka

63. - zadan je uspon ceste od 12 %

- skica :



- a = visinska razlika
- b = horizontalčni pomak

- α - je kut pod kojim se cesta uspinje u u odnosu na horizontali . . .

$\text{uspon ceste u \%} = \frac{\text{visinska razlika}}{\text{horizontalni pomak}}$

$12\ \% = \frac{a}{b}$, kako je $\operatorname{tg}\alpha = \frac{a}{b}$

$\operatorname{tg}\alpha = 12\ \%$

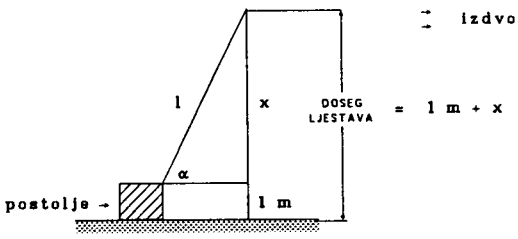
$\operatorname{tg}\alpha = \frac{12}{100} = 0{,}12$

$\operatorname{tg}\alpha = 0{,}12 \quad /\operatorname{tg}^{-1}$

$\alpha = 6^\circ 50' 34''$

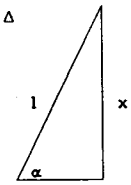
64. zadano : dužina ljestava $l = 30$ m , postolje visine $h = 1$ m , najveći nagib $70°$

skica :



= 1 m + x

izdvojimo pravokutni Δ



$\sin\alpha = \frac{x}{l} \quad / \cdot l$

$x = l \cdot \sin\alpha$

$x = 30 \cdot \sin 70°$

$x = 30 \cdot 0{,}93969 = 28{,}1886$

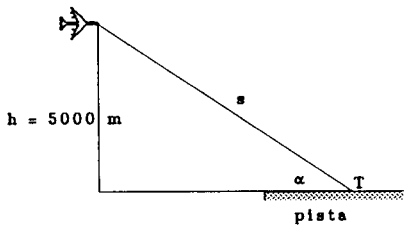
$x = 28{,}19$ m

- pa je doseg ljestava = visina postolj + x = 1 + 28,19 = 29,19 m

65. zadano:

zrakoplov je na visini od 5000 m → $h = 5000$ m , stalni kut α pod kojim se spušta je $15{,}5° \rightarrow \alpha = 15{,}5°$

- koliko je zrakoplov udaljen od točke dodira sa sljetnom pistom → dakle traži se put koji treba preletjeti do točke T u kojoj će dodirnuti pistu . . . iz slike je vidljivo da je hipotenuza pravokutnog trokuta taj traženi put.
- slika:



$\sin\alpha = \frac{h}{s} \quad / \cdot \frac{s}{\sin\alpha}$

$s = \frac{h}{\sin\alpha}$

$s = \frac{5000}{\sin 15{,}5°} = \frac{5000}{0{,}0{,}26724} = 18709{,}77399$ m

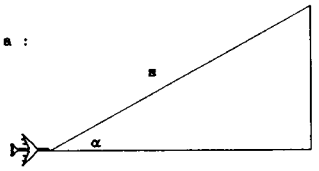
$s = 18709{,}77$ m ili 18,709 km

66. zadano : kut uspinjanja $\alpha = 40°$

brzina zrakoplova $v = 300$ km/h , i vrijeme uspinjanja $t = 10$ sek , traži se visina na koju se popne nakon 10 sek !

$v = 300 \cdot \frac{1000}{3600} = 83{,}3333$ m/sek

- slika :



h = visina

1. izračunamo pređeni put s za t = 10 sek

$s = v \cdot t$

$s = 83{,}3333 \cdot 10 = 8333{,}33$ m

$s = 833.333$ m

2. - visina na koju se popne zrakoplov je h . . .:

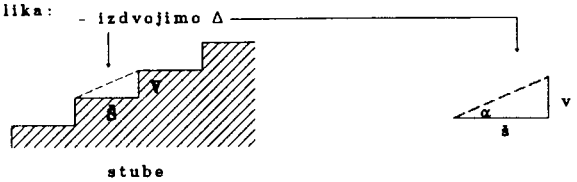
$\sin\alpha = \frac{h}{s} \quad / \cdot s$

$h = s \cdot \sin\alpha = 833{,}333 \cdot \sin 40° = 833{,}333 \cdot 0{,}642788 = 535{,}65645$

$h = 535{,}656$ m $= 536$ m

67. - zaano : visina stube - $v = 14$ cm , i širina stube $š = 30$ cm , kut α uspinjanja stubišta je ?

- slika:



$tg\,\alpha = \frac{v}{š}$

$tg\,\alpha = \frac{14}{30}$

$tg\,\alpha = 0{,}46667 \quad / tg^{-1}$

$\alpha = 25°1'2''$

$\alpha = 25°$

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PO DIJELOVIMA ili po POGLAVLJIMA

ovi POPUSTI vrijede od 02.01.2017. do 21.03.2017.g.

Br.	Naziv ZBIRKE riješenih zadataka	Puna cijena	02.01.17.-11.02.17. 30-60%	12.02.17.-21.03.17. 15-40%
	SREDNJA ŠKOLA			
1.	**Matematika-1-** po **Dakić-Elezović**	740 kn	**333**kn	**435** kn
1.A	ili samo **I –** polugodište (I ,II, III poglavlje)	335 kn	**159** kn	**220** kn
1.E	ili samo NEJEDNADŽBE … 2.8.—4.4..	**200 kn**	**99 kn**	**130 kn**
1.B	**II –** polugodište (IV, V, VII, VIII poglavlje)	465 kn	**199** kn	**325** kn
	ili po djelovima (poglavljima)			
1. C	**Potencije – algebarski izrazi i Algebarski razlomci** Svi zadaci od **1.1.** pa do **2.7.** (320 stranica ...)	**200 kn**	**99 kn**	**120 kn**
1.D	**Jednadžbe , nejednadžbe jednadžbe i nejednadžbe s apsolutnim vrijednostima …** Svi zadaci od **2.8.** pa do **3.5.** (225 stranica ...)	**200 kn**	**99 kn**	**130 kn**
1.E	**Koordinatni sustav u ravnini** Svi zadaci od **4.1.** pa do **4.4.** (250 stranica ...)	**200 kn**	**120 kn**	**140 kn**
10.	**Fizika-1**-po Mikuličić,Varičak,Vernić (mehanika)	240 kn	**99** kn	**99** kn
9.	**Testovi po Dakiću Mat-1**- komplet rješenja - **Zbirke zadataka s pismenih ispita** – Mat-1	555 kn	**315** kn	**360** kn
9.A	ili samo **I –** polugodište (I - II grupa)	200 kn	**111** kn	**150** kn
9.B	**II-** polugodište (III, V, VI, VII grupa)	355 kn	**199** kn	**266** kn
11.	**Matematika -2- - Dakić-Elezović**			
11.A	ili samo **I -** polugodište (I- IV poglavlja)	450 kn	**199** kn	**250** kn
11.B	**II -** polugodište (V - VII poglavlja)	450 kn	**229** kn	**295** kn
	Matematika -2- Dakić-Elezović po dijelovima:			
11.C	**KOMPLEKSNI BROJEVI**	100 kn	**60** kn	**80** kn
11.D	**KVADRATNA JEDNADŽBA**	120 kn	**80** kn	**90** kn
11.E	**POLINOMI 2. STUPNJA**	120 kn	**80** kn	**90** kn
11.F	**TRIGONOMETRIJA pravokutnog trokuta**	140 kn	**80** kn	**90 kn**
12.	**Fizika-2-** po Mikuličić,Varičak,Vernić (toplina i elektricitet)	240 kn	**140** kn	**155** kn
13.	**Testovi po Dakiću Mat-2**- (I-VI grupa)	540 kn	**310** kn	**351** kn

	rješenja - **Zbirke zadataka s pismenih ispita** – Mat-2			
13.A	ili samo **I** – polugodište (I-IV grupa)	340 kn	**199** kn	**238** kn
13.B	**II** – polugodište (V-VI grupa)	200 kn	**150** kn	**170** kn
	ili svako poglavlje – za sebe	155 kn	**50 kn**	**79 kn**
14.	**Matematika-3 - Dakić-Elezović**			
14.A	ili samo **I -** polugodište (trigonometrija)	410 kn	**199** kn	**270** kn
14.B	**II**-polugodište (vektori i analitička geometrija)	440 kn	**239** kn	**289** kn
	Matematika-3 Dakić-Elezović - po **DIJELOVIMA**			
	1. polugodište			
14.A-1	**Trigonometrija 1. dio –** svi zadaci od. 1.1.- 3.3.	210 kn	**99 kn**	**140 kn**
14.A-2	**Trigonometrija 2. dio –** svi zadaci od. 3.3.- 6.5.	210 kn	**99 kn**	**140 kn**
	2. polugodište			
14.C	**VEKTORI –** svi zadaci od. 7.1. – do 7.7.	120 kn	**80** kn	**90** kn

Kod narudžbe matematičkih zbirki riješenih zadataka bitno je naglasiti da li je riječ o gimnazijskom programu ili o tehničkim školama ili matematičkim gimnazijama i o godini izdanja

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Br.	Naziv ZBIRKE	Puna cijena	02.01.17.-11.02.17. 30-60%	12.02.17.-21.03.17. 15-40%
19.	**Fizika-4** po Mikuličić,Varičak,Vernić (atomska i nuklearna)	90 kn	**60** kn	**68** kn
20.	**Matematika-4-** po **Dakić Elezović** za gimnazije (novo izdanje)	775 kn	**375** kn	**503** kn
20.A	ili samo **I –** polugodište (brojevi,nizovi, funkcije)	475 kn	**199** kn	**299** kn
20.B	**II –** polugodište (derivacije, integrali i pr funkcija)	360 kn	**199** kn	**250** kn
	ili po cjelinama ili poglavljima:			
20.D	**1. grupa – BROJEVI – detaljno riješeni svi zadaci 1.1.-1.7. skripta na 190 stranica A4-format**	**175** kn	**99** kn	**120** kn
20.E	**2. grupa –NIZOVI – detaljno riješeni svi zadaci 2.1.-2.7. skripta na 180 stranica A4-format**	**180** kn	**99** kn	**130** kn
20.F	**3. grupa – FUNKCIJE – detaljno riješeni svi zad. 3.1.-3.5. skripta na 180 stranica A4-format**	**180** kn	**99** kn	**130** kn
19.	**Matematika-4-** po **Dakić Elezović** tehničke i matematičke gimnazije	900 kn	**399** kn	**540** kn
19.A	ili samo **I –** polugodište (brojevi,nizovi, funkcije)	475 kn	**199** kn	**299** kn
19.B	**II –** polugodište (derivacije, integrali i primitivna funkcija,dodatak-kombinatorika i vjerojatnost)	460 kn	**299** kn	**322** kn
21.	**Testovi** po **Dakiću Mat-4**-(I-VI grupa) komplet Rješenja zbirke zadataka s pismenih ispita B. Dakić	510 kn	**306** kn	**332** kn
21.A	ili samo **– I** polugodište. (I,II,III poglavlje)	255 kn	**149** kn	**179** kn
21.B	ili samo **– II** polugodište. (IV,V,VI poglavlje)	255 kn	**166** kn	**179** kn

Na ovom letku su ZBIRKE podijeljene po POGLAVLJIMA
isto onako kako vam dolaze u školskim zbirkama
dakle rješenja sada možete kupovati i po poglavljima

Za sve ostalo što nije u ovom cjeniku a treba vam slobodno nas nazovete ili nas kontaktirajte mailom …