1.

1) $\log_5 25 = ?$ Zadatak rješavamo pomoću pravila br.1. : $\log_a b = c \Rightarrow b = a^c$

$\log_5 25 = x$ uvdemo nepoznanicu x , pa imamo $a = 5, b = 25, c = x$

$25 = 5^x$ $b = a^c$, $25 = 5^2$

$5^2 = 5^x$ primjenimo pravilo br.30

$2 = x$

$x = 2$

Pravila za LOGARITMIRANJE

1) $\log_5 25 = x$

$25 = 5^x$

$5^2 = 5^x$

$2 = x$

$x = 2$

2) $\log_3 3 = x$

$3 = 3^x$

$3^1 = 3^x$

$1 = x$

$x = 1$

3) $\log_2 8 = x$

$8 = 2^x$

$2^3 = 2^x$

$3 = x$

$x = 3$

4) $\log_2 8 = x$

$2 = 8^x$

$2^1 = \left(2^3\right)^x$

$2^1 = 2^{3x}$

$1 = 3x$

$3x = 1 \quad / : 3$

$x = \frac{1}{3}$

5) $\log_9 3 = x$

$3 = 9^x$

$3^1 = \left(3^2\right)^x$

$1 = 2x$

$2x = 1 \quad / : 2$

$x = \frac{1}{2}$

6) $\log_4 0.25 = x$

$0.25 = 4^x$

$\frac{25}{100} = 4^x$

$\left(\frac{1}{4}\right) = 4^x$

$4^{-1} = 4^x$

$-1 = x$

$x = -1$

1.

7) $\log 100 = x$

$\log_{10} 100 = x$

$100 = 10^x$

$10^2 = 10^x$

$2 = x$

$x = 2$

8) $\log 0.1 = x$

$\log_{10} 0.1 = x$

$0.1 = 10^x$

$\frac{1}{10} = 10^x$

$10^{-1} = 10^x$

$-1 = x$

$x = -1$

9) $\log 1000 = x$

$\log_{10} 1000 = x$

$1000 = 10^x$

$10^3 = 10^x$

$3 = x$

$x = 3$

10) $\log 0.01 = x$

$\log_{10} 0.01 = x$

$0.01 = 10^x$

$\frac{1}{100} = 10^x$

$\frac{1}{10^2} = 10^x$

$10^{-2} = 10^x$

$-2 = x$

$x = -2$

2. Koliko je:

1) $\log_2 8 = x$

$8 = 2^x$

$2^3 = 2^x$

$3 = x$

$x = 3$

2) $\log_3 27 = x$

$27 = 3^x$

$3^3 = 3^x$

$3 = x$

$x = 3$

3) $\log_2 16 = x$

$16 = 2^x$

$2^4 = 2^x$

$4 = x$

$x = 4$

4) $\log_{\frac{1}{2}} 16 = x$

$16 = \left(\frac{1}{2}\right)^x$

$2^4 = \left(2^{-1}\right)^x$

$2^4 = 2^{-x}$

$4 = -x$

$x = -4$

5) $\log_{\frac{1}{2}} 32 = x$

$32 = \left(\frac{1}{2}\right)^x$

$2^5 = \left(2^{-1}\right)^x$

$2^5 = 2^{-x}$

$5 = -x$

$x = -5$

6) $\log_{\frac{1}{3}} 27 = x$

$27 = \left(\frac{1}{3}\right)^x$

$3^3 = \left(3^{-1}\right)^x$

$3^3 = 3^{-x}$

$3 = -x$

$x = -3$

7) $\log_{\frac{1}{3}} \sqrt{3} = x$

$\sqrt{3} = \left(\frac{1}{3}\right)^x$

$3^{\frac{1}{2}} = \left(3^{-1}\right)^x$

$3^{\frac{1}{2}} = 3^{-x}$

$\frac{1}{2} = -x$

$x = -\frac{1}{2}$

8) $\log_{\frac{1}{2}} 64 = x$

$64 = \left(\frac{1}{2}\right)^x$

$2^6 = \left(2^{-1}\right)^x$

$2^6 = 2^{-x}$

$6 = -x$

$x = -6$

9) $\log_2 \sqrt{2} = x$

$\sqrt{2} = 2^x$

$2^{\frac{1}{2}} = 2^x$

$\frac{1}{2} = x$

$x = \frac{1}{2}$

3.

1) $\log_2 x = 4$

$x = 2^4$

$x = 16$

2) $\log_3 x = 9$

$x = 3^9$

3) $\log_{\frac{1}{2}} x = 2$

$x = \left(\frac{1}{2}\right)^2$

$x = \frac{1}{4}$

4) $\log_9 x = \frac{1}{2}$

$x = (9)^{\frac{1}{2}}$

$x = \sqrt{9}$

$x = 3$

5) $\log_8 x = \frac{1}{3}$

$x = (8)^{\frac{1}{3}}$

$x = \sqrt[3]{8}$

$x = 2$

6) $\log_3 x = \frac{1}{2}$

$x = 3^{\frac{1}{2}}$

$x = \sqrt{3}$

7) $\log_2 x = \frac{1}{3}$

$x = 2^{\frac{1}{3}}$

$x = \sqrt[3]{2}$

8) $\log_2 x = 0$

$x = 2^0$

$x = 1$

9) $\log_{0.1} x = 2$

$x = 0.1^2$

$x = \left(\frac{1}{10}\right)^2$

$x = \frac{1}{100}$

10) $\log_{25} x = \frac{1}{2}$

$x = 25^{\frac{1}{2}}$

$x = \sqrt{25}$

$x = 5$

11) $\log_{27} x = \frac{1}{3}$

$x = 27^{\frac{1}{3}}$

$x = \sqrt[3]{27}$

$x = \sqrt[3]{3^3}$

$x = 3$

4.

1) $\log_x 9 = \frac{2}{3}$

$9 = x^{\frac{2}{3}} \quad /^{\frac{3}{2}}$

$9^{\frac{2}{3}} = x$

$x = 9^{\frac{3}{2}}$

$x = \left(3^2\right)^{\frac{3}{2}}$

$x = 3^{\frac{2}{1}\cdot\frac{3}{2}}$

$x = 3^3$

$x = 27$

2) $\log_x 16 = \frac{4}{5}$

$16 = x^{\frac{4}{5}}$

$16^{\frac{5}{4}} = x$

$x = 16^{\frac{1}{4}\cdot 5}$

$x = \left(\left(2^4\right)^{\frac{1}{4}}\right)^5$

$x = 2^5$

$x = 32$

3) $\log_x 36 = -2$

$36 = x^{-2} \quad /^{-\frac{1}{2}}$

$36^{-\frac{1}{2}} = x$

$x = 36^{-\frac{1}{2}}$

$x = \frac{1}{36^{\frac{1}{2}}}$

$x = \frac{1}{\sqrt{36}}$

$x = \frac{1}{6}$

4) $\log_x 81 = -\frac{4}{3}$

$81 = x^{-\frac{4}{3}} \quad /^{-\frac{3}{4}}$

$81^{-\frac{3}{4}} = x$

$x = 81^{-\frac{3}{4}}$

$x = \left(3^4\right)^{-\frac{3}{4}}$

$x = 3^{\frac{4}{1}\cdot\left(-\frac{3}{4}\right)}$

$x = 3^{-3}$

$x = \frac{1}{3^3}$

$x = \frac{1}{27}$

4.

5) $\log_x \frac{1}{16} = -8$

$\frac{1}{16} = x^{-8} \quad /^{-\frac{1}{8}}$

$\left(\frac{1}{16}\right)^{-\frac{1}{8}} = x$

$x = \left(\frac{1}{16}\right)^{-\frac{1}{8}}$

$x = \left(\frac{16}{1}\right)^{\frac{1}{8}}$

$x = \left(2^4\right)^{\frac{1}{8}}$

$x = 2^{4 \cdot \frac{1}{8}}$

$x = 2^{\frac{1}{2}}$

$x = \sqrt{2}$

6) $\log_x \sqrt{3} = \frac{1}{2}$

$\sqrt{3} = x^{\frac{1}{2}} \quad /^2$

$\sqrt{3^2} = x$

$3 = x$

$x = 3$

7) $\log_x \sqrt[3]{2} = \frac{1}{3}$

$\sqrt[3]{2} = x^{\frac{1}{3}} \quad /^3$

$\sqrt[3]{2^3} = x$

$2 = x$

$x = 2$

8) $\log_x \sqrt[3]{2} = \frac{1}{6}$

$\sqrt[3]{2} = x^{\frac{1}{6}} \quad /^6$

$\sqrt[3]{2^6} = x$

$x = \sqrt[3]{2^6}$

$x = \left(2^6\right)^{\frac{1}{3}}$

$x = 2^{\frac{6}{3}}$

$x = 2^2$

$x = 4$

9) $\log_x 25 = 2$

$25 = x^2 \quad /^{\frac{1}{2}}$

$25^{\frac{1}{2}} = x$

$x = 25^{\frac{1}{2}}$

$x = \sqrt{25}$

$x = 5$

10) $\log_x \frac{8}{27} = 3$

$\frac{8}{27} = x^3$

$\left(\frac{8}{27}\right)^{\frac{1}{3}} = x$

$x = \left(\frac{8}{27}\right)^{\frac{1}{3}}$

$x = \left(\frac{2^3}{3^3}\right)^{\frac{1}{3}}$

$x = \left(\left(\frac{2}{3}\right)^3\right)^{\frac{1}{3}}$

$x = \frac{2}{3}$

5.

1) $3^{\log_3 5} = x \rightarrow$ po pravilu $a^{\log_a x} = x$

2) $2^{\log_2 7} = 7$

3) $\left(\frac{1}{2}\right)^{\log_2 5} = \left(2^{-1}\right)^{\log_2 5} = 2^{-1\log_2 5} = 2^{\log_2 5^{-1}} = 2^{\log_2 \frac{1}{5}} = \frac{1}{5}$

4) $3^{\log_9 6} = 3^{\log_{3^2} 6} = 3^{\frac{1}{2}\log_3 6} = 3^{\log_3 6^{\frac{1}{2}}} = 6^{\frac{1}{2}} = \sqrt{6}$

5) $4^{\log_2 5} = 2^{2\log_2 5} = 2^{\log_2 5^2} = 5^2 = 25$

6) $25^{\log_5 2} = 5^{2\log_5 2} = 5^{\log_5 2^2} = 2^2 = 4$

7) $3^{\log_9 25} = 3^{\log_{3^2} 25} = 3^{\frac{1}{2}\log_3 25} = 3^{\log_3 25^{\frac{1}{2}}} = 25^{\frac{1}{2}} = \sqrt{25} = 5$

8) $\left(\frac{1}{4}\right)^{\log_4 5} = \left(4^{-1}\right)^{\log_4 5} = 4^{\log_4 5^{-1}} = 5^{-1} = \frac{1}{5}$

Pravila za LOGARITMIRANJE

7. Logaritmiraj:

1) $\log\frac{100}{x^2} = \log 100 - \log x^2 = \log 10^2 - 2\log x = 2\log 10 - 2\log x =$

$= 2 \cdot 1 - 2\log x = 2 - 2\log x$

2) $\log\frac{x^2}{10y^3} = \log x^2 - \log\left(10y^3\right) = 2\log x - \left(\log 10 + \log y^3\right) = 2\log x - \log 10 - 3\log y =$

$= 2\log x - 1 - 3\log x$

3) $\log\frac{\sqrt{xy}}{100} = \log\sqrt{xy} - \log 100 = \log\left(\sqrt{x} \cdot \sqrt{y}\right) - \log 10^2 = \log\sqrt{x} + \log\sqrt{y} - 2 \cdot \log 10 =$

$= \log x^{\frac{1}{2}} + \log y^{\frac{1}{2}} - 2 \cdot 1 = \frac{1}{2}\log x + \frac{1}{2}\log y - 2$

4) $\log\frac{\sqrt[3]{x^2}}{100} = \log\sqrt[3]{x^2} - \log 100 = \log\left(x^2\right)^{\frac{1}{3}} - \log 10^2 = \log x^{\frac{2}{3}} - 2\log 10 =$

$= \frac{2}{3} \cdot \log x - 2 \cdot 1 = \frac{2}{3}\log x - 2$

5) $\log\frac{100x^2}{\sqrt{x}} = \log\left(100x^2\right) - \log\sqrt{x} = \log 100 + \log x^2 - \log x^{\frac{1}{2}} =$

$= \log 10^2 + 2\log x - \frac{1}{2}\log x = 2\log 10 + \left(2 - \frac{1}{2}\right)\log x =$

$= 2 + \frac{3}{2}\log x$

7.

6) $\log\dfrac{x^2y^3}{10\sqrt{x}} = \log\left(x^2y^3\right) - \log\left(10\sqrt{x}\right) = \log x^2 + \log y^3 - \left(\log 10 + \log\sqrt{x}\right) =$

$= 2\log x + x + 3\log y - \log 10 - \log x^{\frac{1}{2}} = 2\log x + 3\log y - 1 - \dfrac{1}{2}\log x$

7) $\log\dfrac{x^2y^3}{\sqrt{10}} = \log\left(x^2y^3\right) - \log\sqrt{10} = \log x^2 + \log y^3 - \log 10^{\frac{1}{2}} =$

$= 2\log x + 3\log y - \dfrac{1}{2}\log 10 = 2\log x + 3\log y - \dfrac{1}{2}$

8) $\log\sqrt{\dfrac{10x}{\sqrt{y}}} = \log\dfrac{\sqrt{10x}}{\sqrt{\sqrt{y}}} = \log\dfrac{\sqrt{10}\cdot\sqrt{x}}{\sqrt[4]{y}} = \log\left(\sqrt{10}\cdot\sqrt{x}\right) - \log\sqrt[4]{y} =$

$= \log\sqrt{10} + \log\sqrt{x} - \log y^{\frac{1}{4}} = \log 10^{\frac{1}{2}} + \log x^{\frac{1}{2}} - \dfrac{1}{4}\log y =$

$= \dfrac{1}{2}\log 10 + \dfrac{1}{2}\log x - \dfrac{1}{4}\log y = \dfrac{1}{2} + \dfrac{1}{2}\log x - \dfrac{1}{4\log y}$

9) $\log\sqrt[5]{\dfrac{x^2y^3}{1000}} = \log\left(\dfrac{x^2y^3}{1000}\right)^{\frac{1}{5}} = \dfrac{1}{5}\log\dfrac{x^2y^3}{1000} = \dfrac{1}{5}\left(\log\left(x^2y^3\right) - \log 1000\right) =$

$= \dfrac{1}{5}\left(\log x^2 + \log y^3 - \log 10^3\right) = \dfrac{1}{5}\left(2\log x + 3\log y - 3\log 10\right) =$

$= \dfrac{1}{5}\cdot 2\log x + \dfrac{1}{5}\cdot 3\log y - \dfrac{1}{5}\cdot 3\cdot 1 = \dfrac{2}{5}\log x + \dfrac{3}{5}\log y - \dfrac{3}{5}$

6. Logaritmiraj:

1) $\log(10x) = \log 10 + \log x = 1 + \log x$

2) $\log 100x^2 = \log 100 + \log x^2 = \log 10^2 + 2\log x = 2\log 10 + 2\log x =$
$= 2 \cdot 1 + 2\log x = 2 + 2\log x$

3) $\log\left(x^2 y^3 z^4\right) = \log x^2 + \log y^3 + \log z^4 = 2\log x + 3\log y + 4\log z$

4) $\log \frac{10}{x} = \log 10 - \log x = 1 - \log x$

5) $\log \frac{x^2}{y^3} = \log x^2 - \log y^3 = 2\log x - 3\log y$

6) $\log x^2 = 2\log x$

7) $\log x^3 = 3\log x$

8) $\log \frac{x^3}{y^5} = \log x^3 - \log y^5 = 3\log x - 5\log y$

9) $\log \frac{\sqrt{x}}{y^3} = \log \sqrt{x} - \log y^3 = \log x^{\frac{1}{2}} - 3\log y = \frac{1}{2}\log x - 3\log y$

10) $\log \sqrt[3]{x} = \log x^{\frac{1}{3}} = \frac{1}{3}\log x$

6.

11) $\log\sqrt[5]{x} = \log x^{\frac{1}{5}} = \frac{1}{5}\log x$

12) $\log\sqrt[4]{\frac{10}{x}} = \log\frac{\sqrt[4]{10}}{\sqrt[4]{x}} = \log\sqrt[4]{10} - \log\sqrt[4]{x} = \log 10^{\frac{1}{4}} - \log x^{\frac{1}{4}} = \frac{1}{4}\log 10 - \frac{1}{4}\log x =$

$= \frac{1}{4}\cdot 1 - \frac{1}{4}\log x = \frac{1}{4} - \frac{1}{4}\log x$

13) $\log\sqrt[3]{100x^2} = \log\left(100x^2\right)^{\frac{1}{3}} = \frac{1}{3}\log\left(100\cdot x^2\right) = \frac{1}{3}\left(\log 100 + \log x^2\right) =$

$= \frac{1}{3}\cdot\left(\log 10^2 + 2\log x\right) = \frac{1}{3}\cdot\left(2\log 10 + 2\log x\right) = \frac{1}{3}\left(2\cdot 1 + 2\log x\right) =$

$= \frac{2}{3} + \frac{2}{3}\log x$

14) $\log\sqrt[5]{10x^3} = \log\left(10x^3\right)^{\frac{1}{5}} = \frac{1}{5}\log\left(10x^3\right) = \frac{1}{5}\left(\log 10 + \log x^3\right) =$

$= \frac{1}{5}\left(1 + 3\log x\right) = \frac{1}{5} + \frac{3}{5}\log x$

15) $\log 1 = 0 \quad \rightarrow \quad$ po pravilu $\quad \log_a 1 = 0$

16) $\log_3 1 = 0$

17) $\log_5 1 = 0$

8.

1) $\dfrac{\log 36}{\log 2+\log 3}=\dfrac{\log 6^2}{\log(2\cdot 3)}=\dfrac{2\log 6}{\log 6}=\dfrac{2\cancel{\log 6}}{\cancel{\log 6}}=2$

2) $\dfrac{\log 324}{\log 3+\log 6}=\dfrac{\log 18^2}{\log(3\cdot 6)}=\dfrac{2\log 18}{\log 18}=\dfrac{2\cancel{\log 18}}{\cancel{\log 18}}=2$

3) $\dfrac{\log 3-\log 5}{\log\frac{9}{25}}=\dfrac{\log\frac{3}{5}}{\log\left(\frac{3}{5}\right)^2}=\dfrac{\log\frac{3}{5}}{2\log\frac{3}{5}}=\dfrac{1\cdot\cancel{\log\frac{3}{5}}}{2\cdot\cancel{\log\frac{3}{5}}}=\dfrac{1}{2}$

4) $\dfrac{\log\frac{4}{25}}{\log 2-\log 5}=\dfrac{\log\left(\frac{2}{5}\right)^2}{\log\frac{2}{5}}=\dfrac{2\cdot\log\frac{2}{5}}{\log\frac{2}{5}}=\dfrac{2\cdot\cancel{\log\frac{2}{5}}}{\cancel{\log\frac{2}{5}}}=2$

5) $\dfrac{1+\log 2.5}{\log 35-\log 7}=\dfrac{\log 10+\log 2.5}{\log\frac{35}{7}}=$

$=\dfrac{\log(10\cdot 2.5)}{\log 5}=\dfrac{\log 25}{\log 5}=\dfrac{\log 5^2}{\log 5}=\dfrac{2\log 5}{\log 5}=\dfrac{2\cancel{\log 5}}{\cancel{\log 5}}=2$

Pravila za LOGARITMIRANJE

9.

1) $\log(x+2)+\log(x-4)=\log(2x-3)$

uvjeti :

$x+2>0 \quad x-4>0 \quad 2x-3>0$

$x>-2 \quad x>4 \quad 2x>3$

$x>\frac{3}{2}$

$\Downarrow$

$x>4$

$\log\left[(x+2)(x-4)\right]=\log(2x-3)$

$(x+2)(x-4)=2x-3$

$x^2-4x+2x-8-2x+3=0$

$x^2-4x-5=0$

$$x_{1,2}=\frac{-(-4)\pm\sqrt{(-4)^2-4\cdot 1\cdot(-5)}}{2\cdot 1}=\frac{4\pm\sqrt{16+20}}{2}=\frac{4\pm\sqrt{36}}{2}=\frac{4\pm 6}{2}$$

$$x_1=\frac{4-6}{2}=-\frac{2}{2} \qquad x_2=\frac{4+6}{2}=\frac{10}{2}$$

$x_1=-1 \qquad x_2=5$

uvjet kaže da mora biti: $x>4$ pa je jedino rješenje $x=5$

2) $\log(x+3)+\log(x-1)=\log(5x+7)$

uvjeti:

$x+3>0 \quad x-1>0 \quad 5x+7>0$

$x>-3 \quad x>1 \quad 5x>-7$

$x>-\frac{7}{5}$

$\Downarrow$

uvijet je $x>1$

$\log\left[(x+3)\cdot(x-1)\right]=\log(5x+7)$

$(x+3)\cdot(x-1)=5x+7$

$x^2-x+3x-3-5x-7=0$

$x^2-3x-10=0$

$$x_{1,2}=\frac{-(-3)\pm\sqrt{(-3)^2-4\cdot 1\cdot(-10)}}{2\cdot 1}=\frac{3\pm\sqrt{9+40}}{2}=\frac{3\pm\sqrt{49}}{2}=\frac{3\pm 7}{2}$$

$$x_1=\frac{3-7}{2}=-\frac{4}{2} \qquad x_2=\frac{3+7}{2}=\frac{10}{2}$$

$x_1=-2 \qquad x_2=5$

uvjet kaže da mora biti: $x>1$

$\Downarrow$

pa je: $x=5$ jedino rješenje

3) $\log(x-2)+\log(x+3)=\log(4x+2)$

4) $\log x+\log(x+2)=\log(7x-6)$

10.

1.) $\dfrac{3}{\log x+2}=2-\log x$

$\log x=t$

$\dfrac{3}{t+2}=2-t \quad /\cdot(t+2)$

$3=(2-t)(t+2)$

$3=2^2-t^2$

$3=4-t^2$

$3-4=-t^2$

$-1=-t^2$

$t^2=1 \quad /\sqrt{\ }$

$t=\pm\sqrt{1}$

$t_1=-1 \qquad t_2=1$

$\log x=t$

$\log x=-1$ $\qquad$ $\log x=1$

$x=10^{-1}$ $\qquad$ $x=10^{1}$

$x=\dfrac{1}{10}$ $\qquad$ $x=10$

uvjet: $x\neq\dfrac{1}{100}$

pa su rješenja: $x_1=\dfrac{1}{10}$ i $x_2=10$

uvjet:

$\log x+2\neq 0$

$\log x\neq -2$

$x\neq 10^{-2}$

$x\neq\dfrac{1}{100}$

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