

\*\*\*\* MLADEN SRAGA \*\*\*\*  
2010.

UNIVERZALNA ZBIRKA  
POTPUNO RIJEŠENIH ZADATAKA  
PRIRUČNIK ZA SAMOSTALNO UČENJE

# **MATEMATIKA**

# **3**

# **TRIGONOMETRIJA**

## **1. DIO**

**M.I.M.-SRAGA**  
 $\sqrt{\alpha}$

Crtaње grafa funkcije  $f(x) = a \sin(bx + c)$

1. odredimo  $a$ ,  $b$ ,  $c$
2. odredimo prvu nul točku  $N_1 = -\frac{c}{b}$
3. odredimo PERIOD  $P = \frac{2\pi}{b}$
4. odredimo treću nul točku  $N_3 = N_1 + P$
5. odredimo drugu nul točku  $N_2 = \frac{N_1 + N_3}{2}$
6. ako je  $a > 0$  tada je  $x_M = \frac{N_1 + N_2}{2}$       ako je  $a < 0$  tada je  $x_m = \frac{N_1 + N_2}{2}$
7. ako je  $a > 0$  tada je  $x_m = \frac{N_2 + N_3}{2}$       ako je  $a < 0$  tada je  $x_M = \frac{N_2 + N_3}{2}$
8. amplituda je  $= a$       povučemo pravce  $y = a$  i  $y = -a$
9. crtamo graf tako da naznačimo sve tri nul toče  $x_M$  i  $x_m$  ....

primjer :

Nacrtaj graf :

$$f(x) = 3 \sin\left(\frac{x}{2} + \frac{\pi}{2}\right)$$

1.  $a = 3, b = \frac{1}{2}, c = \frac{\pi}{2}$

2.  $N_1 = -\frac{c}{b} = -\frac{\frac{\pi}{2}}{\frac{1}{2}} = -\frac{2\pi}{2} = -\pi$

3.  $P = \frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = \frac{2\pi}{1} = 4\pi$

4.  $N_3 = N_1 + P = -\pi + 4\pi = 3\pi$

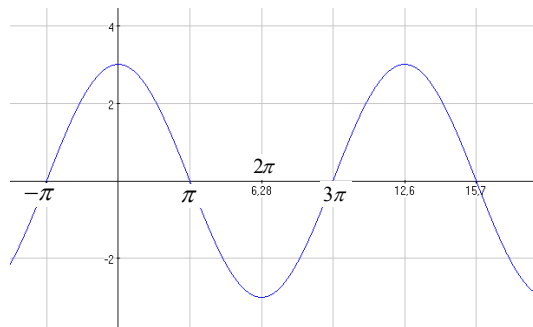
5.  $N_2 = \frac{N_1 + N_3}{2} = \frac{-\pi + 3\pi}{2} = \frac{2\pi}{2} = \pi$

6.  $a > 0$  tada je  $x_M = \frac{N_1 + N_2}{2} = \frac{-\pi + \pi}{2} = \frac{0}{2} = 0$

7.  $a > 0$  tada je  $x_m = \frac{N_2 + N_3}{2} = \frac{\pi + 3\pi}{2} = \frac{4\pi}{2} = 2\pi$

8.  $y = 3$ ,  $y = -3$

slika:  $\Rightarrow$



još jednom taj isti zadatak:

Nacrtaj graf :

$$f(x)=3 \sin \left( \frac{x}{2} + \frac{\pi}{2} \right)$$

$$1. \quad a=3, \quad b=\frac{1}{2}, \quad c=\frac{\pi}{2}$$

$$2. \quad N_1 = -\frac{c}{b} = -\frac{\frac{\pi}{2}}{\frac{1}{2}} = -\frac{2\pi}{2} = -\pi$$

$$3. \quad P = \frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = \frac{2\pi}{\frac{1}{2}} = 4\pi$$

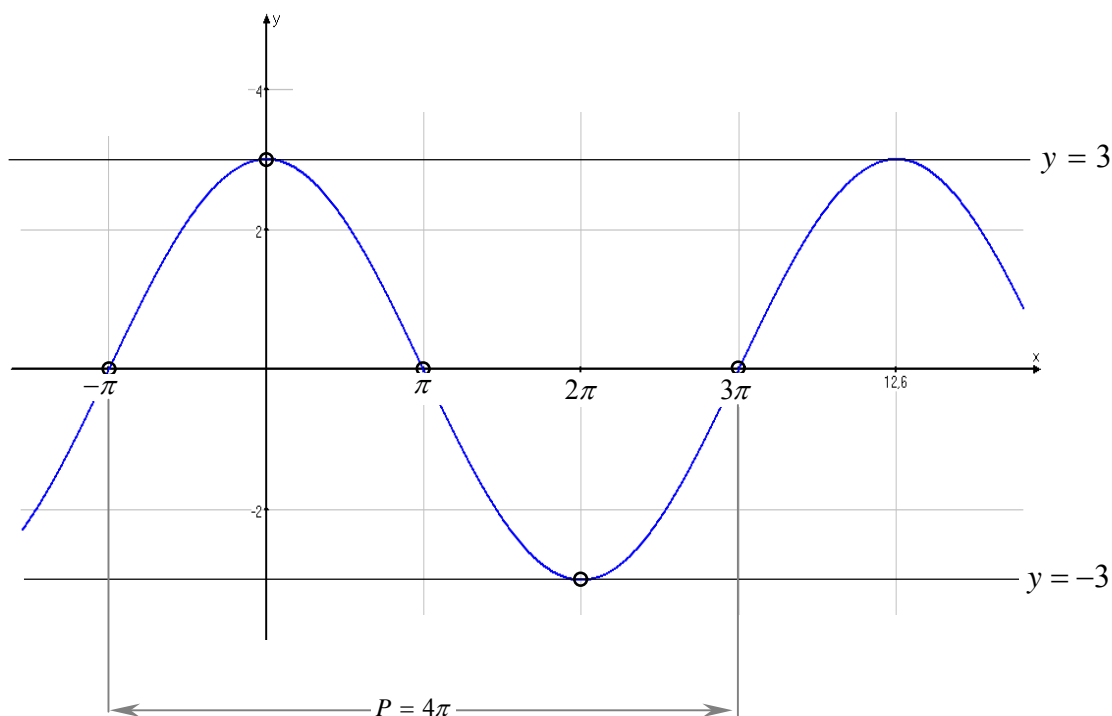
$$4. \quad N_3 = N_1 + P = -\pi + 4\pi = 3\pi$$

$$5. \quad N_2 = \frac{N_1 + N_3}{2} = \frac{-\pi + 3\pi}{2} = \frac{2\pi}{2} = \pi$$

$$6. \quad a > 0 \quad \text{tada je} \quad x_M = \frac{N_1 + N_2}{2} = \frac{-\pi + \pi}{2} = \frac{0}{2} = 0$$

$$7. \quad a > 0 \quad \text{tada je} \quad x_m = \frac{N_2 + N_3}{2} = \frac{\pi + 3\pi}{2} = \frac{4\pi}{2} = 2\pi$$

$$8. \quad y = 3, \quad y = -3$$



Nacrtaj graf :

$$f(x) = 2 \sin\left(\frac{x}{2} - \frac{\pi}{2}\right)$$

1.  $a = 2, b = \frac{1}{2}, c = -\frac{\pi}{2}$

2.  $N_1 = -\frac{c}{b} = -\frac{-\frac{\pi}{2}}{\frac{1}{2}} = \frac{2\pi}{2} = \pi$

3.  $P = \frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = \frac{2\pi}{1} = 4\pi$

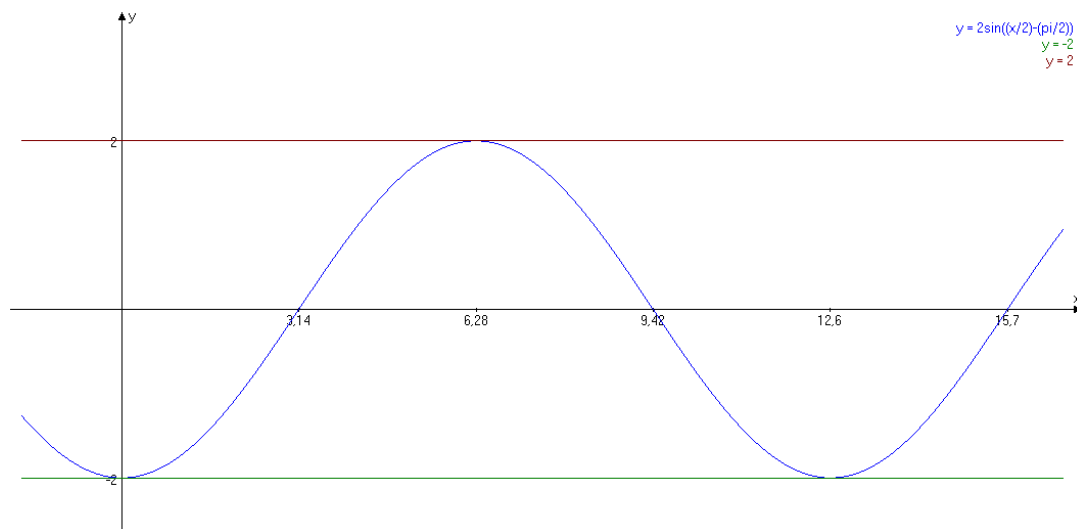
4.  $N_3 = N_1 + P = \pi + 4\pi = 5\pi$

5.  $N_2 = \frac{N_1 + N_3}{2} = \frac{\pi + 5\pi}{2} = \frac{6\pi}{2} = 3\pi$

6.  $a > 0$  tada je  $x_M = \frac{N_1 + N_2}{2} = \frac{\pi + 3\pi}{2} = \frac{4\pi}{2} = 2\pi$

7.  $a > 0$  tada je  $x_m = \frac{N_2 + N_3}{2} = \frac{3\pi + 5\pi}{2} = \frac{8\pi}{2} = 4\pi$

8.  $y = 2, y = -2$



Nacrtaj graf :

$$f(x) = \frac{3}{2} \sin\left(2x + \frac{\pi}{4}\right)$$

$$1. \quad a = \frac{3}{2}, \quad b = 2, \quad c = \frac{\pi}{4}$$

$$2. \quad N_1 = -\frac{c}{b} = -\frac{\frac{\pi}{4}}{2} = -\frac{\frac{\pi}{4}}{\frac{2}{1}} = -\frac{\pi}{8}$$

$$3. \quad P = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$$

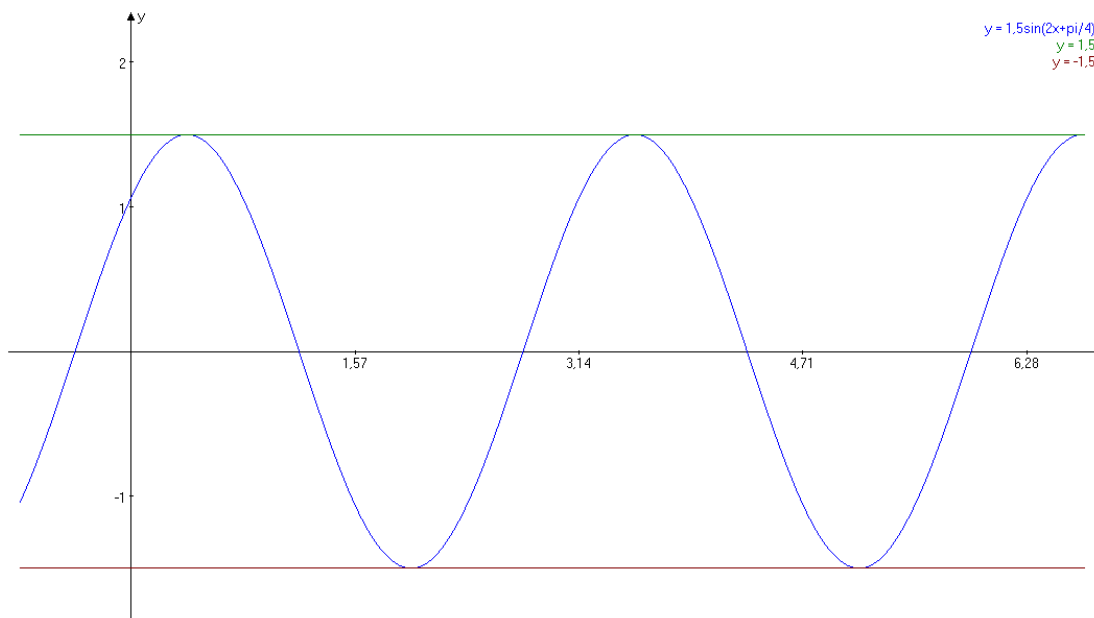
$$4. \quad N_3 = N_1 + P = -\frac{\pi}{8} + \pi = \frac{7\pi}{8}$$

$$5. \quad N_2 = \frac{N_1 + N_3}{2} = \frac{-\frac{\pi}{8} + \frac{7\pi}{8}}{2} = \frac{\frac{6\pi}{8}}{2} = \frac{\frac{3\pi}{4}}{\frac{2}{1}} = \frac{3\pi}{8}$$

$$6. \quad a > 0 \quad \text{tada je} \quad x_M = \frac{N_1 + N_2}{2} = \frac{-\frac{\pi}{8} + \frac{3\pi}{8}}{2} = \frac{\frac{2\pi}{8}}{2} = \frac{\frac{\pi}{4}}{\frac{2}{1}} = \frac{\pi}{8}$$

$$7. \quad a > 0 \quad \text{tada je} \quad x_m = \frac{N_2 + N_3}{2} = \frac{\frac{3\pi}{8} + \frac{7\pi}{8}}{2} = \frac{\frac{10\pi}{8}}{2} = \frac{5\pi}{4} = 4\pi$$

$$8. \quad y = 2, \quad y = -2$$



Zadatak iz školske zbirke: poglavlje 4.1.

Zad. 6. pod 4)

Nacrtaj graf :

$$f(x) = -\frac{1}{2} \sin\left(4x - \frac{2\pi}{3}\right)$$

1.  $a = -\frac{1}{2}, b = 4, c = -\frac{2\pi}{3}$

2.  $N_1 = -\frac{c}{b} = -\frac{-\frac{2\pi}{3}}{4} = \frac{\frac{2\pi}{3}}{4} = \frac{\pi}{6}$

3.  $P = \frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$

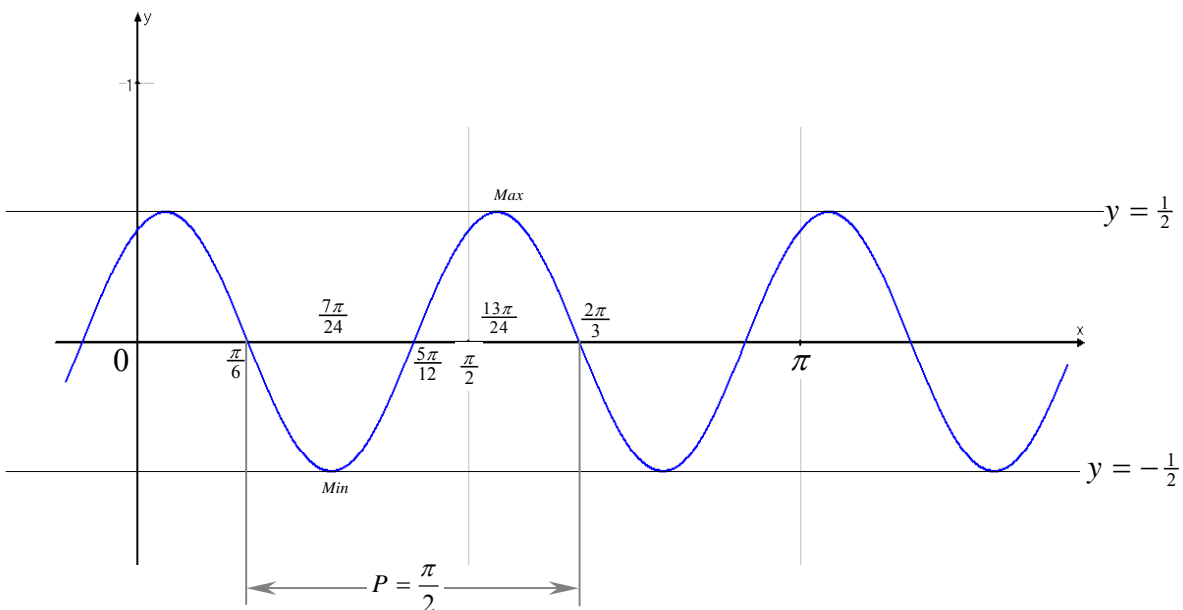
4.  $N_3 = N_1 + P = \frac{\pi}{6} + \frac{\pi}{2} = \frac{2\pi}{3}$

5.  $N_2 = \frac{N_1 + N_3}{2} = \frac{\frac{\pi}{6} + \frac{2\pi}{3}}{2} = \frac{\frac{5\pi}{6}}{2} = \frac{5\pi}{12}$

6.  $a < 0$  tada je  $x_m = \frac{N_1 + N_2}{2} = \frac{\frac{\pi}{6} + \frac{5\pi}{12}}{2} = \frac{\frac{7\pi}{12}}{2} = \frac{7\pi}{24}$

7.  $a < 0$  tada je  $x_M = \frac{N_2 + N_3}{2} = \frac{\frac{5\pi}{12} + \frac{2\pi}{3}}{2} = \frac{\frac{13\pi}{12}}{2} = \frac{13\pi}{24}$

8.  $y = \frac{1}{2}, y = -\frac{1}{2}$



Zadatak iz školske zbirke: poglavlje 4.1.

Zad. 7. pod 2)

Nacrtaj graf :

$$f(x) = 2 \cos\left(3x + \frac{\pi}{2}\right) \quad \text{koristimo formulu redukcije: } \cos x = \sin\left(x + \frac{\pi}{2}\right)$$

$$f(x) = 2 \cos\left(3x + \frac{\pi}{2}\right) = 2 \sin\left(\left(3x + \frac{\pi}{2}\right) + \frac{\pi}{2}\right) = 2 \sin\left(3x + \frac{\pi}{2} + \frac{\pi}{2}\right) = 2 \sin(3x + \pi)$$

i tako smo "cos" preveli u "sin"

$$f(x) = 2 \cos\left(3x + \frac{\pi}{2}\right) = 2 \sin(3x + \pi) \quad \text{i sada crtamo tu funkciju već objašnjenim postupkom:}$$

$$f(x) = 2 \sin(3x + \pi)$$

1.  $a = 2, \quad b = 3, \quad c = \pi$

2.  $N_1 = -\frac{c}{b} = -\frac{\pi}{3}$

3.  $P = \frac{2\pi}{b} = \frac{2\pi}{3}$

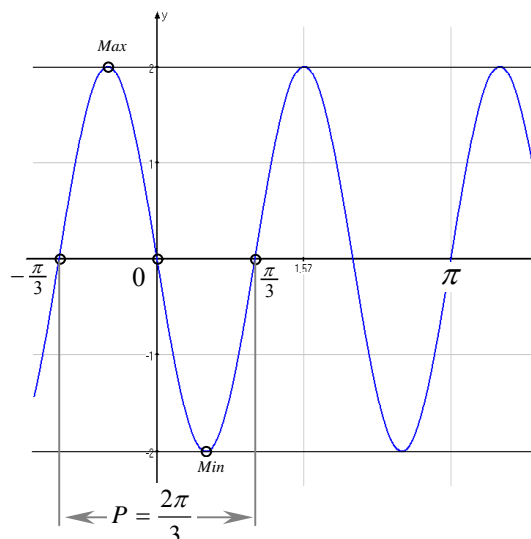
4.  $N_3 = N_1 + P = -\frac{\pi}{3} + \frac{2\pi}{3} = \frac{\pi}{3}$

5.  $N_2 = \frac{N_1 + N_3}{2} = \frac{-\frac{\pi}{3} + \frac{\pi}{3}}{2} = \frac{0}{2} = 0$

6.  $a < 0$  tada je  $x_m = \frac{N_1 + N_2}{2} = \frac{-\frac{\pi}{3} + 0}{2} = \frac{-\frac{\pi}{3}}{2} = -\frac{\pi}{6}$

7.  $a < 0$  tada je  $x_M = \frac{N_2 + N_3}{2} = \frac{0 + \frac{\pi}{3}}{2} = \frac{\frac{\pi}{3}}{2} = \frac{\pi}{6}$

8.  $y = 2, \quad y = -2$



Matematika-3 - univerzalna zbirka potpuno riješenih zadataka za treći razred srednje škole

Crtanje grafa trigonometrijskih funkcija

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