M-4. Ako za kompleksni broj z vrijedi $\frac{z}{1-2i}-\frac{2\overline{z}}{1+2i}=5i$, onda je $|z|$ jednak

A. $\frac{5\sqrt{5}}{3}$ B. $\frac{6\sqrt{6}}{7}$ C. $\frac{10\sqrt{10}}{7}$ D. $\frac{3\sqrt{3}}{2}$ E. $2\sqrt{2}$

$\frac{z}{1-2i}-\frac{2\overline{z}}{1+2i}=5i$ Uzimamo da je: $z=x+yi$, $\overline{z}=x-yi$

$$\frac{z}{1-2i}-\frac{2\overline{z}}{1+2i}=5i \qquad z=x+yi \ , \ \overline{z}=x-yi$$

$$\frac{x+yi}{1-2i}-\frac{2(x-yi)}{1+2i}=5i \qquad /\cdot(1-2i)\cdot(1+2i)$$

$$(x+yi)\cdot(1+2i)-2(x-yi)\cdot(1-2i)=5i\cdot(1-2i)\cdot(1+2i)$$

$$x+2xi+yi+2yi^2-2\left(x-2xi-yi+2yi^2\right)=5i\cdot\left(1^2-2^2i^2\right) \qquad \boxed{i^2=-1}$$

$$x+2xi+yi+2y\cdot(-1)-2\cdot\left(x-2xi-yi+2y\cdot(-1)\right)=5i\cdot\left(1-4\cdot(-1)\right)$$

$$x+2xi+yi-2y-2x+4xi+2yi-2\cdot(-2y)=5i\cdot(1+4)$$

$$x-2x-2y+4y+2xi+4xi+yi+2yi=25i$$

$$-x+2y+6xi+3yi=25i$$

$$(2y-x)+(6x+3y)i=0+25i$$

Ako je: $\mathrm{Re}_1+\mathrm{Im}_1=\mathrm{Re}_2+\mathrm{Im}_2$

Tada je: $\mathrm{Re}_1=\mathrm{Re}_2$ i $\mathrm{Im}_1=\mathrm{Im}_2$

$\mathrm{Re}_1=\mathrm{Re}_2$ i $\mathrm{Im}_1=\mathrm{Im}_2$

$2y-x=0$ $\qquad 6x+3y=25$

$2y=x \rightarrow$ $\qquad 6\cdot 2y+3y=25$

$$12y+3y=25$$

$$15y=25$$

$$y=\frac{25}{15}$$

$$y=\frac{5}{3} \qquad x=2y=2\cdot\frac{5}{3}$$

$$x=\frac{10}{3}$$

$$|z|=\sqrt{x^2+y^2}$$

$$|z|=\sqrt{\left(\frac{10}{3}\right)^2+\left(\frac{5}{3}\right)^2}=\sqrt{\frac{100}{9}+\frac{25}{9}}=\sqrt{\frac{125}{9}}=\frac{\sqrt{125}}{\sqrt{9}}=\frac{\sqrt{25\cdot 5}}{3}=\frac{5\sqrt{5}}{3}$$

$$|z|=\frac{5\sqrt{5}}{3}$$