Zadaci 5.3.

9. koristimo pravilo: $\log_a b = c \rightarrow b = a^c$

1.) $\log_x 8 = -\frac{3}{4}$

$x^{-\frac{3}{4}} = 8 \quad / \; -\frac{4}{3}$

$x = 8^{-\frac{4}{3}}$

$x = (2^3)^{-\frac{4}{3}}$

$x = 2^{3 \cdot (-\frac{4}{3})}$

$x = 2^{-4}$

$x = \frac{1}{2^4}$

$x = \frac{1}{16}$

kako se vidi iz zadatka odredimo a,b,c i samo postavimo $a^c = b$

jednadžbu potenciramo RECIPROČNOM vrijednosti potencije na nepoznanici x

u ovom zadatku potencija na x je $-\frac{3}{4}$

a njena recipročna vrijednost je $-\frac{4}{3}$

jer sada se potencija sa x pomnoži sa potencijom kojom potenciramo jednadžbu . . .

i $-\frac{3}{4} \cdot \left(-\frac{4}{3}\right) = 1$ uvijek dobijemo 1

pa nam tako nepoznanica x ima eksp. $= x^1$

pa je ljeva strana jednadžbe gotova , desno je obično potenciranje poznate baze sa nekim eksponentom po već poznatim pravilima . . .

2.) $\log_x \frac{1}{8} = -\frac{3}{2}$

$x^{-\frac{3}{2}} = \frac{1}{8} \quad / \; -\frac{2}{3}$

$x = \left(\frac{1}{8}\right)^{-\frac{2}{3}}$

$x = \left(\frac{1}{2^3}\right)^{-\frac{2}{3}}$

$x = (2^{-3})^{-\frac{2}{3}}$

$x = 2^{-3 \cdot (-\frac{2}{3})}$

$x = 2^2$

$x = 4$

3) $\log_x 64 = -3$

$x^{-3} = 64 \quad / \; -\frac{1}{3}$

$x = 64^{-\frac{1}{3}}$

$x = (2^6)^{-\frac{1}{3}}$

$x = 2^{-\frac{6}{3}}$

$x = 2^{-2}$

$x = \frac{1}{2^2}$

$x = \frac{1}{4}$

4) $\log_x 8 = 1{,}5$

$x^{1,5} = 8$

$x^{\frac{3}{2}} = 2^3 \quad / \; \frac{2}{3}$

$x = (2^3)^{\frac{2}{3}}$

$x = 2^{3 \cdot \frac{2}{3}}$

$x = 2^2$

$x = 4$

9.

5.3.

5) $\log_x \frac{1}{16} = -4$

$x^{-4} = \frac{1}{16} \quad / ^{-\frac{1}{4}}$

$x = \left(\frac{1}{16}\right)^{-\frac{1}{4}}$

$x = \left(\frac{1}{2^4}\right)^{-\frac{1}{4}}$

$x = \left(2^{-4}\right)^{-\frac{1}{4}}$

$x = 2^{-4\cdot\left(-\frac{1}{4}\right)}$

$x = 2$

$x = 2^1$

$x = 2$

6) $\log_x \sqrt{2} = \frac{1}{4}$

$x^{\frac{1}{4}} = \sqrt{2} \quad / ^4$

$x = \sqrt{2}^{\,4}$

$x = \left(2^{\frac{1}{2}}\right)^4$

$x = 2^{\frac{4}{2}}$

$x = 2^2$

$x = 4$

7) $\log_x 27 = -\frac{3}{4}$

$x^{-\frac{3}{4}} = 27 \quad / ^{-\frac{4}{3}}$

$x = 27^{-\frac{4}{3}}$

$x = \left(3^3\right)^{-\frac{4}{3}}$

$x = 3^{3\cdot\left(-\frac{4}{3}\right)}$

$x = 3$

$x = 3^{-4}$

$x = \frac{1}{3^4}$

$x = \frac{1}{81}$

8) $\log_x 0{,}125 = -2$

$x^{-2} = 0{,}125 \quad / ^{-\frac{1}{2}}$

$x = \left(\frac{125}{1000}\right)^{-\frac{1}{2}}$

$x = \left(\frac{1}{8}\right)^{-\frac{1}{2}}$

$x = 8^{\frac{1}{2}}$

$x = \sqrt{8}$

$x = \sqrt{4\cdot 2}$

$x = \sqrt{4}\cdot\sqrt{2}$

$x = 2\sqrt{2}$

9) $\log_x \frac{7}{3} = 1$

$\frac{7}{3} = x^1$

$x = \frac{7}{3}$

10) $\log^x \frac{8}{27} = -3$

$\frac{8}{27} = x^{-3} \quad / ^{-\frac{1}{3}}$

$\left(\frac{8}{27}\right)^{-\frac{1}{3}} = x$

$x = \left(\frac{2^3}{3^3}\right)^{-\frac{1}{3}} = \left(\left(\frac{2}{3}\right)^3\right)^{-\frac{1}{3}} = \left(\frac{2}{3}\right)^{3\cdot\left(-\frac{1}{3}\right)} = \left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$

10.

koristimo pravilo: $a^{\log_a x} = x$

1) $5^{\log_5 10} = 10$

2) $\left(\frac{1}{5}\right)^{\log_5 10} = \left(5^{-1}\right)^{\log_5 10} = 5^{-1\log_5 10} = 5^{\log_5 10^{-1}} = 10^{-1} = \frac{1}{10}$

3) $3^{\log_3 3} = 3$

4) $\left(\frac{1}{3}\right)^{\log_3 11} = \left(3^{-1}\right)^{\log_3 11} = 3^{-1\log_3 11} = 3^{\log_3 11^{-1}} = 11^{-1} = \frac{1}{11}$

5) $3^{2\log_9 12} = 3^{\log_9 12^2} = 3^{\log_{3^2} 12^2} = 3^{\frac{1}{2}\log_3 12^2} = 3^{\log_3 (12^2)^{1/2}} =$

$= (12^2)^{\frac{1}{2}} = \sqrt{12^2} = 12$

6) $2^{2\log_4 7} = 2^{\log_4 7^2} = 2^{\log_{2^2} 7^2} = 2^{\frac{1}{2}\log_2 7^2} = 2^{\log_2 (7^2)^{\frac{1}{2}}} =$

$= (7^2)^{\frac{1}{2}} = \sqrt{7^2} = 7$

7) $4^{\log_2 3} = 2^{2\log_2 3} = 2^{\log_2 3^2} = 3^2 = 9$

8) $3^{-2\log_9 20} = 3^{\log_9 20^{-2}} = 3^{\log_{3^2} 20^{-2}} = 3^{\frac{1}{2}\log_3 20^{-2}} = 3^{\log_3 (20^{-2})^{\frac{1}{2}}} =$

$= (20^{-2})^{\frac{1}{2}} = \sqrt{20^{-2}} = \sqrt{\left(\frac{1}{20}\right)^2} = \frac{1}{20}$

11.

1) $100^{-\log\frac{5}{2}} = 100^{\log\left(\frac{5}{2}\right)^{-1}} = (10^2)^{\log\left(\frac{5}{2}\right)^{-1}} = 10^{2\log\left(\frac{5}{2}\right)^{-1}} =$

$= 10^{\log\left[\left(\frac{5}{2}\right)^{-1}\right]^2} = 10^{\log\left(\frac{5}{2}\right)^{-2}} = \left(\frac{5}{2}\right)^{-2} = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$

5.3.

11.

2)
$$81^{\frac{1}{2}\log_3 7} = 81^{\log_3 (7)^{\frac{1}{2}}} = (3^4)^{\log_3 (7)^{\frac{1}{2}}} = 3^{4\log_3 (7)^{\frac{1}{2}}} =$$
$$= 3^{\log_3 \left[7^{\frac{1}{2}}\right]^4} = \left[7^{\frac{1}{2}}\right]^4 = 7^{\frac{4}{2}} = 7^2 = 49$$

3)
$$8^{-\log_4 9} = 8^{\log_4 9^{-1}} = (2^3)^{\log_4 9^{-1}} = 2^{3\log_{2^2} 9^{-1}} =$$
$$= 2^{\log_{2^2} (9^{-1})^3} = 2^{\frac{1}{2}\log_2 (9^{-1})^3} = 2^{\log_2 \left[(9^{-1})^3\right]^{\frac{1}{2}}} = \left[(9^{-1})^3\right]^{\frac{1}{2}} =$$
$$= (9^{-3})^{\frac{1}{2}} = \left[\left(\frac{1}{9}\right)^3\right]^{\frac{1}{2}} = \sqrt{\left(\frac{1}{9}\right)^3} = \sqrt{\frac{1}{729}} = \frac{\sqrt{1}}{\sqrt{729}} = \frac{1}{27}$$

4)
$$9^{-\log_3 8} = 9^{\log_3 8^{-1}} = (3^2)^{\log_3 8^{-1}} = 3^{2\log_3 8^{-1}} = 3^{\log_3 (8^{-1})^2} =$$
$$= (8^{-1})^2 = \left(\frac{1}{8}\right)^2 = \frac{1}{64}$$

5)
$$\left(\frac{1}{2}\right)^{2\log_2 \frac{1}{2}} = \left(\frac{1}{2}\right)^{\log_2 \left(\frac{1}{2}\right)^2} = (2^{-1})^{\log_2 \frac{1}{4}} = 2^{-1\log_2 \frac{1}{4}} = 2^{\log_2 \left(\frac{1}{4}\right)^{-1}} =$$
$$= \left(\frac{1}{4}\right)^{-1} = 4$$

6)
$$\left(\frac{1}{4}\right)^{-2\log_8 125} = \left(\frac{1}{4}\right)^{\log_8 (125)^{-2}} = (4^{-1})^{\log_{2^3} (125^{-2})} = 4^{-1\log_{2^3} (125^{-2})} =$$
$$= 4^{\log_{2^3} (125^{-2})^{-1}} = 4^{\frac{1}{3}\log_2 125^2} = 4^{\log_2 (125^2)^{\frac{1}{3}}} = (2^2)^{\log_2 (125^2)^{\frac{1}{3}}} =$$
$$= 2^{2\log_2 (125^{\frac{2}{3}})} = 2^{\log_2 (125^{\frac{2}{3}})^2} = \left(125^{\frac{2}{3}}\right)^2 = 125^{\frac{4}{3}} = \sqrt[3]{125^4} =$$
$$= \sqrt[3]{125^3 \cdot 125} = 125\sqrt[3]{125} = 125 \cdot 5 = 625$$

12.

$f(x) = \log_5 x$

$f(1) = \log_5 1 = 1$

$f(-2) = \log_5 (-2)$ nije definirano

$f(0.2) = \log_5 0.2 = \log_5 \frac{2}{10}^{1} = \log_5 \frac{1}{5} = \log_5 5^{-1} = -1$

$f(125) = \log_5 125 = \log_5 5^3 = 3$

$f(0.04) = \log_5 0.04 = \log_5 \frac{4}{100}^{1}{}_{25} = \log_5 \frac{1}{25} = \log_5 25^{-1} =$

$= \log_5 (5^2)^{-1} = \log_5 5^{-2} = -2$

13.

$f(x) = \log_{\frac{1}{4}} x$

$f(2) = \log_{\frac{1}{4}} 2 = \log_{4^{-1}} 2 = -\frac{1}{1}\log_4 2 = \log_4 2^{-1} = \log_{2^2} 2^{-1} =$

$= \frac{1}{2}\log_2 2^{-1} = \log_2 (2^{-1})^{\frac{1}{2}} = \log_2 2^{-\frac{1}{2}} = -\frac{1}{2}$

$f(-4) = \log_{\frac{1}{4}} (-4)$ nije definirano

$f(0.25) = \log_{\frac{1}{4}} 0.25 = \log_{\frac{1}{4}} \frac{25}{100}^{1}{}_{4} = \log_{\frac{1}{4}} \frac{1}{4} = 1$

$f(0) = \log_{\frac{1}{4}} 0$ nije definirano

$f(0.5) = \log_{\frac{1}{4}} 0.5 = \log_{\frac{1}{4}} \frac{5}{10}^{1}{}_{2} = \log_{\frac{1}{4}} \frac{1}{2} = \log_{4^{-1}} \frac{1}{2} = -\frac{1}{1}\log_4 \frac{1}{2} =$

$= \log_4 \left(\frac{1}{2}\right)^{-1} = \log_{2^2} \left(\frac{1}{2}\right)^{-1} = \frac{1}{2}\log_2 \left(\frac{1}{2}\right)^{-1} = \log_2 \left[\left(\frac{1}{2}\right)^{-1}\right]^{\frac{1}{2}} =$

$= \log_2 \left(\frac{1}{2}\right)^{-\frac{1}{2}} = \log_2 (2^{-1})^{-\frac{1}{2}} = \log_2 2^{\frac{1}{2}} = \frac{1}{2}$

14.

$f(x) = \log x$

$f(0.1) = \log 0.1 = \log \frac{1}{10} = \log 10^{-1} = -1$

$f(100) = \log 100 = \log 10^2 = 2$

$f(0.001) = \log 0.001 = \log \frac{1}{1000} = \log 1000^{-1} = \log (10^3)^{-1} =$

$= \log 10^{-3} = -3$

$f(10^{-5}) = \log 10^{-5} = -5$

14. (nastavak)

$$f(0.01^{-4}) = \log 0.01^{-4} = \log \left(\frac{1}{100}\right)^{-4} = \log (100^{-1})^{-4} =$$

$$= \log 100^4 = \log (10^2)^4 = \log 10^8 = 8$$

15.

$$\lfloor \log 123 \rfloor = \lfloor 2,089 \rfloor = 2$$

$$\lfloor \log 5,5 \rfloor = \lfloor 0,740 \rfloor = 0$$

$$\lfloor \log 0,989 \rfloor = \lfloor -4.80 \cdot 10^{-3} \rfloor = \lfloor -0,0048 \rfloor = -1$$

$$\lfloor \log 0,01 \rfloor = \lfloor -2 \rfloor = -2$$

16.

$$\lfloor \log_2 77 \rfloor = \left\lfloor \frac{\log 77}{\log 2} \right\rfloor = \left\lfloor \frac{1,88649}{0,30103} \right\rfloor = \lfloor 6,266 \rfloor = 6$$

$$\lfloor \log_3 0,1 \rfloor = \left\lfloor \frac{\log 0,1}{\log 3} \right\rfloor = \left\lfloor \frac{-1}{0.47712} \right\rfloor = \lfloor -2.095 \rfloor = -3$$

$$\lfloor \log_8 11111 \rfloor = \left\lfloor \frac{\log 11111}{\log 8} \right\rfloor = \left\lfloor \frac{4,04575}{0.90309} \right\rfloor = \lfloor 4,479 \rfloor = 4$$

$$\lfloor \log_{\frac{1}{4}} 25 \rfloor = \left\lfloor \frac{\log 25}{\log \frac{1}{4}} \right\rfloor = \left\lfloor \frac{1.39794}{-0,60206} \right\rfloor = \lfloor -2.321 \rfloor = -3$$

$$\lfloor \log_{\frac{1}{5}} 0,01 \rfloor = \left\lfloor \frac{\log 0,01}{\log \frac{1}{5}} \right\rfloor = \left\lfloor \frac{-2}{-0,69897} \right\rfloor = \lfloor 2.861 \rfloor = 2$$

Ovo je jako mali dio potpuno riješenih zadataka iz naše interne skripte ili zbirke potpuno riješenih zadataka koju koristimo u radu našeg centra – poduka i online poduka.
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