2. KVADRATNE JEDNADŽBE

2.1. Kvadratna jednadžba

	Oblici jednadžbe	Riješenja
br.1	$ax^2+bx+c=0$	$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
br.2	$ax^2+bx=0 \quad c=0$	$x_1=0\ ,\ x_2=-\frac{b}{a}$
br.3	$ax^2+c=0 \quad b=0$	$x_1=+\sqrt{-\frac{c}{a}}\ ,\ x_2=-\sqrt{-\frac{c}{a}}$

Primjeri:

Ovaj zadatak možemo rješavati na dva načina ili po br.3 ili načinu koji ću prvo pokazati . . .

sada taj isti zadatak riješen po br.3

1.) $4x^2-25=0$

$4x^2=25 \quad /:4$

$x^2=\frac{25}{4} \quad /\sqrt{\ }$

$x_{1,2}=\pm\sqrt{\frac{25}{4}}$

$x_{1,2}=\pm\frac{\sqrt{25}}{\sqrt{4}}$

$x_{1,2}=\pm\frac{5}{2}$

$x_1=\frac{5}{2} \qquad x_2=-\frac{5}{2}$

$4x^2-25=0$

$a=4\ ,\ b=0\ ,\ c=-25$

$x_1=+\sqrt{-\frac{c}{a}}\ ,\ x_2=-\sqrt{-\frac{c}{a}}$

$x_1=\sqrt{-\frac{-25}{4}} \qquad x_2=-\sqrt{-\frac{-25}{4}}$

$x_1=\sqrt{\frac{25}{4}} \qquad x_2=-\sqrt{\frac{25}{4}}$

$x_1=\frac{\sqrt{25}}{\sqrt{4}} \qquad x_2=-\frac{\sqrt{25}}{\sqrt{4}}$

$x_1=\frac{5}{2} \qquad x_2=-\frac{5}{2}$

iz dobivenih riješenja vidimo da obadva načina daju iste rezultate po mojim iskustvima češće se koristi prvi način

prvi način

2.) $x^2+1=0$

$x^2=-1 \quad /\sqrt{\ }$

$x_{1,2}=\pm\sqrt{-1} \qquad \sqrt{-1}=i$

$x_{1,2}=\pm i$

$x_1=i \qquad x_2=-i$

drugi način

$x^2+1=0$

$a=1\ ,\ b=0\ ,\ c=1$

$x_1=+\sqrt{-\frac{c}{a}}\ ,\ x_2=-\sqrt{-\frac{c}{a}}$

$x_1=\sqrt{-\frac{1}{1}} \qquad x_2=-\sqrt{-\frac{1}{1}}$

$x_1=\sqrt{-1} \qquad x_2=-\sqrt{-1}$

$x_1=i \qquad x_2=-i$

dalje radim samo po prvom načinu

3.) $x^2-3=0$

$x^2=3 \quad /\sqrt{\ }$

$x_{1,2}=\pm\sqrt{3}$

$x_1=\sqrt{3} \qquad x_2=\sqrt{3}$

4.) $\frac{x^2}{3}-0{,}48=0 \quad /\cdot 3$

$x^2-1{,}44=0$

$x^2=1{,}44 \quad /\sqrt{\ }$

$x_{1,2}=\pm\sqrt{1{,}44}$

$x_{1,2}=\pm 1{,}2$

$x_1=1{,}2 \qquad x_2=-1{,}2$

5.) $4x^2-0{,}81=0$

$4x^2=0{,}81$

$4x^2=\frac{81}{100} \quad /:4$

$x^2=\frac{81}{400} \quad /\sqrt{\ }$

$x_{1,2}=\pm\sqrt{\frac{81}{400}}=\pm\frac{\sqrt{81}}{\sqrt{400}}$

$x_{1,2}=\pm\frac{9}{20}$

$x_1=\frac{9}{20} \qquad x_2=-\frac{9}{20}$

6.) $2x^2+1=0$

$2x^2=-1 \quad /:$

$x^2=-\frac{1}{2} \quad /\sqrt{\ }$

$x_{1,2}=\pm\sqrt{-\frac{1}{2}}=\pm\sqrt{\frac{1}{2}\cdot(-1)}=$

$=\pm\sqrt{\frac{1}{2}}\cdot\sqrt{-1}=\pm\sqrt{\frac{1}{2}}\cdot i$

$x_{1,2}=\pm\sqrt{\frac{1}{2}}\cdot i=\pm\frac{\sqrt{2}}{2}i$

$x_1=\frac{\sqrt{2}}{2}i \qquad x_2=-\frac{\sqrt{2}}{2}i$

$$\sqrt{\frac{1}{2}}=\frac{\sqrt{1}}{\sqrt{2}}=\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{\sqrt{2^2}}=\frac{\sqrt{2}}{2}$$

U ovom dokumentu imate 15 –stranica sa potpuno riješenim zadacima po školskoj zbirci od autora : Dakić-Elezović – izdavač ELEMENTd.o.o.

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3.

1) $3x^2 + x = 0$

$x(3x+1) = 0$

$x = 0$ $\quad$ $3x+1=0$

$x_1 = 0$ $\quad$ $3x = -1 \,/ : 3$

$x = -\frac{1}{3}$ $\quad$ $x_2 = -\frac{1}{3}$

2) $\frac{1}{2}x^2 - \frac{2}{3}x = 0$

$x\left(\frac{1}{2}x - \frac{2}{3}\right) = 0$

$x = 0$ $\quad$ $\frac{1}{2}x - \frac{2}{3} = 0$

$x_1 = 0$ $\quad$ $\frac{1}{2}x = \frac{2}{3} \,/ \cdot 2$

$x = \frac{4}{3}$

$x_2 = \frac{4}{3}$

3) $x^2\sqrt{2} - x\sqrt{8} = 0$

$x\left(x\sqrt{2} - \sqrt{8}\right) = 0$

$x = 0$ $\quad$ $x\sqrt{2} - \sqrt{8} = 0$

$x_1 = 0$ $\quad$ $x\sqrt{2} = \sqrt{8} \,/ : \sqrt{2}$

$x = \frac{\sqrt{8}}{\sqrt{2}}$

$x = \sqrt{\frac{8}{2}}$

$x = \sqrt{4}$

$x = 2$

$x_2 = 2$

4) $\frac{1}{2}x^2 + 6x = 0$

$x\left(\frac{1}{2}x + 6\right) = 0$

$x = 0$ $\quad$ $\frac{1}{2}x + 6 = 0$

$x_1 = 0$ $\quad$ $\frac{1}{2}x = -6 \,/ \cdot 2$

$x = -12$

5) $\frac{2}{3}x^2 - \frac{3}{4}x = 0$

$x\left(\frac{2}{3}x - \frac{3}{4}\right) = 0$

$x = 0$ $\quad$ $\frac{2}{3}x - \frac{3}{4} = 0$

$x_1 = 0$ $\quad$ $\frac{2}{3}x = \frac{3}{4} \,/ \cdot \frac{3}{2}$

$x = \frac{9}{8}$

$x_2 = \frac{9}{8}$

6) $x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0$ $\quad$ $x - 1 = 0$

$x_1 = 0$ $\quad$ $x = 1$

$x_2 = 1$

7) $0{,}04x^2 + 5x = 0$

$x(0{,}04x + 5) = 0$

$x = 0$ $\quad$ $0{,}04x + 5 = 0$

$x_1 = 0$ $\quad$ $0{,}04x = -5 \,/ : 0{,}04$

$x = -\frac{5}{0{,}04}$

$x = -125$

$x_2 = -125$

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2.2. Rješenja kvadratne jednadžbe

1.

Primjeri:

ZA RAZUMIJEVANJE OVOG ZADATKA I SAMO RJEŠAVANJE POTREBNO JE DOBRO POZNAVANJE ALGEBARSKIH IZRAZA TO JE GRADIVO SA POČETKA PRVOG RAZREDA

1.) $x^2 - 2x - 3 = 0$ sada kažemo $A^2 = x^2$ i $2AB = -2x$ iz te dvije pretpostavke izračunamo A i B

$\underbrace{x^2}_{A^2} \underbrace{- 2x}_{2AB}$

$A^2 = x^2 \quad / \sqrt{\ }$

$A = x$ sada to uvrstimo u $2AB = -2x$

$2xB = -2x \quad / : 2x$

$B = -1$

dobili smo $A = x$ i $B = -1$, postavimo $(A + B)^2$

$(A + B)^2 = (x - 1)^2 = x^2 - 2x + 1$ prva dva člana su ista kao i u početku jednadžbe ali treći član (+ 1) je broj koji trebamo dodati i oduzeti od početne jednadžbe

dodam i oduzmem jedan $-1 + 1 = 0$ to ne mijenja jednakost

pišemo $\underbrace{x^2 - 2x + 1}\underbrace{- 1 - 3} = 0$

sada prva tri člana prikažem kao kvadrat binoma a preostala dva člana zbrojim

$(x - 1)^2 - 4 = 0$

$(x - 1)^2 - 2^2 = 0 \rightarrow$ kada se bolje pogleda ljeva strana jednakosti vidiljivo je da je to razlika kvadrata

$(x - 1 - 2) \cdot (x - 1 + 2) = 0$

$(x - 3) \cdot (x + 1) = 0$

$x - 3 = 0 \qquad x + 1 = 0$

$x_1 = 3 \qquad x_2 = -1$

2.) $x^2 - 4x - 5 = 0$

$A^2 = x^2 \quad / \sqrt{\ }$

$A = x \rightarrow 2AB = -4x$

$2xB = -4x \quad / : 2x$

$A = x \qquad B = -2$

$(x - 2)^2 = x^2 - 4x + 4 \rightarrow$ dodajemo i oduzimamo 4

$x^2 - 4x - 5 = 0$

$\underbrace{x^2 - 4x + 4}\underbrace{- 4 - 5} = 0$

$(x - 2)^2 - 9 = 0$

$A^2 - B^2$

$(x - 2)^2 - 3^2 = 0$

$(A - B) \cdot (A + B)$

$(x - 2 - 3) \cdot (x - 2 + 3) = 0$

$(x - 5) \cdot (x + 1) = 0$

$x - 5 = 0 \qquad x + 1 = 0$

$x = 5 \qquad x = -1$

$x_1 = 5 \qquad x_2 = -1$

3.) $x^2 - 6x + 13 = 0$

$A^2 = x^2 \quad / \sqrt{\ }$

$A = x \qquad 2AB = -6x$

$2xB = -6x \quad / : 2x$

$B = -3$

$(x - 3)^2 = x^2 - 6x + 9$

dodajemo i oduzimamo 9

$x^2 - 6x + 9 - 9 + 13 = 0$

$(x - 3)^2 + 4 = 0$

$(x - 3)^2 = -4 \quad / \sqrt{\ }$

$\sqrt{(x - 3)^2} = \sqrt{-4}$

$x - 3 = \pm\sqrt{4} \cdot \sqrt{-1}$

$x - 3 = \pm 2i$

$x - 3 = 2i \qquad x - 3 = -2i$

$x = 3 + 2i \qquad x = 3 - 2i$

$x_1 = 3 + 2i \qquad x_2 = 3 - 2i$

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Zadaci:

1) $x^2-4x+3=0$

$A^2=x^2 /\sqrt{\ }$ $\rightarrow$ $2AB=-4x$

$A=x$ $\quad 2\cdot xB=-4x /:2x$

$B=-2$

$(x-2)^2=x^2-4x+4$

dodajemo i oduzimamo 4

$x^2-4x+3=0$

$\underbrace{x^2-4x+4}\ \underbrace{-4+3}=0$

$(x-2)^2 \quad -1=0$

$A^2 \quad - B^2 =$

$(x-2-1)\cdot(x-2+1)=0$

$(x-3)\ (x-1)=0$

$x-3=0 \qquad x-1=0$

$x=3 \qquad x=1$

$x_1=3 \qquad x_2=1$

2) $x^2+2x-8=0$

$A^2=x^2/\sqrt{\ }$ $\quad 2AB=2x$

$A=x$ $\quad 2xB=2x/:2x$

$B=1$

$(x+1)^2=x^2+2x+1$

dodajemo i oduzimamo 1

$x^2+2x-8=0$

$\underbrace{x^2+2x+1}\ \underbrace{-1-8}=0$

$(x+1)^2 \quad -9=0$

$(x+1)^2-3^2=0$

$(x+1-3)\ (x+1+3)=0$

$(x-2)\ (x+4)=0$

$x-2=0 \qquad x+4=0$

$x=2 \qquad x=-4$

$x_1=2 \qquad x_2=-4$

3) $4x^2-4x-3=0$

$A^2=4x^2/\sqrt{\ }$ $\quad 2AB=-4x$

$A=2x$ $\quad 2\cdot 2xB=-4x$

$4xB=-4x /:4x$

$B=-1$

$(2x-1)^2=4x^2-4x+1$

dodajemo i oduzimamo 1

$4x^2-4x-3=0$

$\underbrace{4x^2-4x+1}-1-3=0$

$(2x-1)^2-4=0$

$(2x-1)^2-2^2=0$

$(2x-1-2)(2x-1+2)=0$

$(2x-3)(2x+1)=0$

$2x-3=0 \qquad 2x+1=0$

$2x=3/:2 \qquad 2x=-1/:2$

$x=\frac{3}{2} \qquad x=-\frac{1}{2}$

$x_1=\frac{3}{2} \qquad x_2=-\frac{1}{2}$

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2.

1) $6x^2 - x - 2 = 0$

$a = 6 \quad b = -1 \quad c = -2$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 6 \cdot (-2)}}{2 \cdot 6}$$
$$= \frac{1 \pm \sqrt{1 + 48}}{12}$$
$$= \frac{1 \pm \sqrt{49}}{12}$$
$$= \frac{1 \pm 7}{12}$$

$x_1 = \frac{1-7}{12} \qquad x_2 = \frac{1+7}{12}$

$x_1 = \frac{-6}{12} \qquad x_2 = \frac{8}{12}$

$x_1 = -\frac{1}{2} \qquad x_2 = \frac{2}{3}$

2) $10x^2 + 3x - 4 = 0$

$a = 10 \quad b = 3 \quad c = -4$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3 \pm \sqrt{3^2 - 4 \cdot 10 \cdot (-4)}}{2 \cdot 10}$$
$$= \frac{-3 \pm \sqrt{9 + 160}}{20}$$
$$= \frac{-3 \pm \sqrt{169}}{20}$$
$$= \frac{-3 \pm 13}{20}$$

$x_1 = \frac{-3-13}{20} \qquad x_2 = \frac{-3+13}{20}$

$x_1 = \frac{-16}{20} \qquad x_2 = \frac{10}{20}$

$x_1 = \frac{-4}{5} \qquad x_2 = \frac{1}{2}$

3) $27x^2 - 3x - 4 = 0$

$a = 27 \quad b = -3 \quad c = -4$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 27 \cdot (-4)}}{2 \cdot 27}$$
$$= \frac{3 \pm \sqrt{9 + 432}}{54}$$
$$= \frac{3 \pm \sqrt{441}}{54}$$
$$= \frac{3 \pm 21}{54}$$

$x_1 = \frac{3+21}{54} \qquad x_2 = \frac{3-21}{54}$

$x_1 = \frac{24}{54} \qquad x_2 = \frac{-18}{54}$

$x_1 = \frac{4}{9} \qquad x_2 = \frac{-1}{3}$

4) $5x^2 - 2x - 3 = 0$

$a = 5 \quad b = -2 \quad c = -3$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 5 \cdot (-3)}}{2 \cdot 5}$$
$$= \frac{2 \pm \sqrt{4 + 60}}{10}$$
$$= \frac{2 \pm \sqrt{64}}{10}$$
$$= \frac{2 \pm 8}{10}$$

$x_1 = \frac{2-8}{10} \qquad x_2 = \frac{2+8}{10}$

$x_1 = -\frac{6}{10} \qquad x_2 = \frac{10}{10}$

$x_1 = -\frac{3}{5} \qquad x_2 = 1$

7. 5)

7) $\underbrace{(a^2-b^2)}_{a}x^2 \underbrace{-4ab}_{b}x \underbrace{-a^2+b^2}_{c}=0$

$$x_{1,2}=\frac{4ab \pm\sqrt{16a^2b^2-4\cdot(a^2-b^2)\cdot(-a^2+b^2)}}{2(a^2-b^2)}$$

$$=\frac{4ab \pm\sqrt{16a^2b^2+4(a^2-b^2)\cdot(a^2-b^2)}}{2(a^2-b^2)}$$

$$=\frac{4ab + \sqrt{16a^2b^2+4(a^2-b^2)^2}}{2(a^2-b^2)}$$

$$=\frac{4ab \pm\sqrt{16a^2b^2+4(a^4-2a^2b^2+b^4)}}{2(a^2-b^2)}$$

$$=\frac{4ab \pm\sqrt{16a^2b^2+4a^4-8a^2b^2+4b^4}}{2(a^2-b^2)}$$

$$=\frac{4ab \pm\sqrt{4a^4+8a^2b^2+4b^4}}{2(a^2-b^2)}$$

$$=\frac{4ab \pm\sqrt{4(a^4+2a^2b^2+b^4)}}{2(a^2-b^2)}=\frac{4ab\pm 2(a^2+b^2)}{2(a^2-b^2)}$$

6)

$$x_1=\frac{4ab+2(a^2+b^2)}{2(a^2-b^2)}=\frac{\cancel{2}(2ab+a^2+b^2)}{\cancel{2}(a^2-b^2)}=\frac{a^2+2ab+b^2}{a^2-b^2}$$

$$=\frac{(a+b)^{\cancel{2}}}{(a-b)\cancel{(a+b)}}=\frac{a+b}{a-b}$$

$$x_2=\frac{4ab-2(a^2+b^2)}{2(a^2-b^2)}=\frac{\cancel{2}(2ab-a^2-b^2)}{\cancel{2}(a^2-b^2)}=\frac{-(a^2-2ab+b^2)}{(a-b)(a+b)}$$

$$=\frac{-(a-b)^{\cancel{2}}}{\cancel{(a-b)}(a+b)}=\frac{-(a-b)}{a+b}=\frac{-a+b}{a+b}$$

$= -a+2b$

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8. 1) $x^3 - 1 = 0$

$(x-1)(x^2 + x \cdot 1 + 1^2) = 0$

$x - 1 = 0 \qquad x^2 + x + 1 = 0$

$x_1 = 1 \qquad a = 1,\; b = 1,\; c = 1$

$$x_{2,3} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{-1 \pm \sqrt{-3}}{2}$$

$$x_{2,3} = \frac{-1 \pm i\sqrt{3}}{2}$$

$x_1 = 1 \qquad x_{2,3} = \dfrac{-1 \pm i\sqrt{3}}{2}$

2) $x^3 + 27 = 0 \qquad x^3 + 3^3 = 0$

$(x+3)(x^2 - 3x + 3^2) = 0$

$x + 3 = 0 \qquad x^2 - 3x + 3^2 = 0$

$x_1 = -3 \qquad x^2 - 3x + 9 = 0$

$a,\; b,\; c$

$$x_{2,3} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$$

$$= \frac{3 \pm \sqrt{9 - 36}}{2} = \frac{3 \pm \sqrt{-27}}{2} = \frac{3 \pm 3i\sqrt{3}}{2}$$

$x_1 = -3 \qquad x_{2,3} = \dfrac{3 \pm 3i\sqrt{3}}{2}$

3) $m^3 - 64 = 0 \qquad m^3 - 4^3 = 0$

$(m-4)(m^2 + 4m + 16) = 0$

$m - 4 = 0 \qquad m^2 + 4m + 16 = 0$

$m_1 = 4 \qquad a,\; b,\; c$

$$m_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 16}}{2 \cdot 1}$$

$$= \frac{-4 \pm \sqrt{16 - 64}}{2} = \frac{-4 \pm \sqrt{48}}{2}$$

$$= \frac{-4 \pm \sqrt{-16 \cdot 3}}{2} = \frac{-4 \pm 4\sqrt{-3}}{2}$$

$$= \frac{\cancel{2}(-2 + 2\sqrt{-3})}{\cancel{2}} = -2 + 2\sqrt{-3}$$

$$m_{2,3} = -2 + 2i\sqrt{3}$$

$m_1 = 4 \qquad m_{2,3} = -2 + 2i\sqrt{3}$

2.3. Diskriminanta kvadratne jednadžbe

DISKRMINANTA $D = b^2 - 4ac$ kada je

- $D > 0$ x_1, x_2 su različiti realni brojevi
- $D = 0$ $x_1 = x_2$ isti realni brojevi
- $D < 0$ x_1, x_2 su konjugirano kompleksni brojevi
- $D \geq 0$ x_1, x_2 su realni brojevi

Primjeri:

1.) $3x^2 - 2x + m = 0$

nema realna riješenja

to znači da su riješenja konjugirano kompleksni brojevi pa je $D < 0$

$a = 3,\ b = -2,\ c = m$

$D < 0$

$b^2 - 4ac < 0$

$(-2)^2 - 4 \cdot 3 \cdot m < 0$

$4 - 12m < 0$

$-12m < -4 \quad / : (-12)$

$m > \frac{4}{12}$

$m > \frac{1}{3}$

dakle za $m > \frac{1}{3}$ nema realne korjene

2.) $mx^2 - 6x + 1 = 0$ ima realne korjene $\rightarrow$ to znači da je $D \geq 0$

$a = m,\ b = -6,\ c = 1$

$D \geq 0$

$b^2 - 4ac \geq 0$

$(-6)^2 - 4 \cdot m \cdot 1 \geq 0$

$36 - 4m \geq 0$

$-4m \geq -36 \quad / (-4)$

$m \leq 9$

dakle za $m \leq 9$ jed. ima realne korjene

3.) $2x^2 - (m-2)x - m = 0$

ima oba korjena realna i jednaka $\rightarrow$ $D = 0$

$a = 2,\ b = -(m-2),\ c = -m$

$D = 0$

$b^2 - 4ac = 0$

$\left(-(m-2)\right)^2 - 4 \cdot 2 \cdot (-m) = 0$

$(m-2)^2 + 8m = 0$

$m^2 - 4m + 4 + 8m = 0$

$m^2 + 4m + 4 = 0$

$(m+2)^2 = 0 \quad / \sqrt{\ }$

$\sqrt{(m+2)^2} = 0$

$m + 2 = 0$

$m = -2$

dakle za $m = -2$ oba korjena jednadžbe su realna i jednaka

4.) $2mx^2 - x - 1 = 0$

ima oba korijena realna i jednaka

to znači da je $D = 0$

$a = 2m,\ b = -1,\ c = -1$

$D = 0$

$b^2 - 4ac = 0$

$(-1)^2 - 4 \cdot 2m \cdot (-1) = 0$

$1 + 8m = 0$

$8m = -1 \quad / : 8$

$m = -\frac{1}{8}$

za $m = -\frac{1}{8}$ jed. ima oba korijena realna i jednaka

5.) $x^2 - 2mx + 2 = 0$

nema realnih korijena

to znači da je $D < 0$

$a = 1,\ b = -2m,\ c = 2$

$D < 0$

$b^2 - 4ac < 0$

$(-2m)^2 - 4 \cdot 1 \cdot 2 < 0$

$4m^2 - 8 < 0$

$4m^2 - 8 < 0 \ / : 4$

$m^2 - 2 < 0$

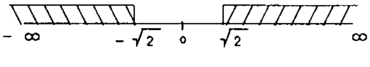
$m^2 - \sqrt{2}^2 < 0$

$(m - \sqrt{2}) \cdot (m + \sqrt{2}) < 0$

1.

$m - \sqrt{2} > 0 \qquad m + \sqrt{2} < 0$

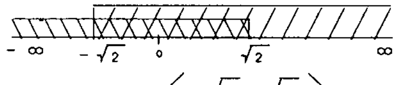
$m > \sqrt{2} \qquad m < -\sqrt{2}$



2.

$m - \sqrt{2} < 0 \qquad m + \sqrt{2} > 0$

$m < \sqrt{2} \qquad m > -\sqrt{2}$



$m \in \left\langle -\sqrt{2}, \sqrt{2} \right\rangle$

6.) $x^2 - 2mx + m^2 - 1 = 0$

ima realne i različite korijene

to znači da je $D > 0$

$a = 1,\ b = -2m,\ c = m^2 - 1$

$D > 0$

$b^2 - 4ac > 0$

$(-2m)^2 - 4 \cdot 1 \cdot (m^2 - 1) > 0$

$4m^2 - 4m^2 + 4 > 0$

$4 > 0$ tvrdnja je istinita dakle za svaki $m \in R$ jednadžba ima različite realne korjene jer $D = 4$ za svaki $m \in R$ pa m nema utjecaj na vrijednost D

zadatke riješio Mladen Sraga

1.

1.) $x^2 + 6x + 4 = 0$

$a = 1 \quad b = 6 \quad c = 4$

$D = b^2 - 4ac$

$D = 6^2 - 4 \cdot 1 \cdot 4 = 36 - 16 = 20$

$\underline{\underline{D = 20}}$, $D > 0 \Rightarrow$ RJEŠENJA SU RAZLIČITI REALNI BROJEVI

2.) $9x^2 + 3x + 5 = 0$

$a = 9 \quad b = 3 \quad c = 5$

$D = b^2 - 4ac$

$D = 3^2 - 4 \cdot 9 \cdot 5 = 9 - 180$

$\underline{\underline{D = -171}}$ $\quad D < 0 \Rightarrow$ RJEŠENJA SU KONJUGIRANO KOMPLEKSNI BROJEVI

3.) $4t^2 - 5t + 2 = 0$

$a = 4 \quad b = -5 \quad C = 2$

$D = b^2 - 4ac$

$D = (-5)^2 - 4 \cdot 4 \cdot 2$

$D = 25 - 32$

$D = -7$, $D < 0 \Rightarrow$ RJEŠENJA SU KONJUGIRANO KOMPLEKSNI BROJEVI

ČITAJ: $\rightarrow$ D JE MANJE OD NULE

2.

1) $\dfrac{x^2+4x+c=0}{c=?}$

jednadžba ima jednaka rješenja

$D=0$

$D=b^2-4ac$

$x^2+4x+c=0$

$a=1 \quad b=4 \quad c=c$

$b^2-4ac=0$

$4^2-4\cdot1\cdot c=0$

$16-4c=0$

$16=4c \,/:4$

$c=4$

2) $\dfrac{2x^2-x+c=0}{a=2 \quad b=-1 \quad c=c}$

Različita i realna rješenja

$D>0$

$D>0$

$b^2-4ac>0$

$(-1)^2-4\cdot2\cdot c>0$

$1-8c>0$

$-8c>-1 \,/:(-8)$

$c<\frac{1}{8}$

3) $\dfrac{x^2-5x+c-1=0}{a=1 \quad b=-5 \quad c=c-1}$

Nema realnih rješenja

$D<0$

$D<0$

$b^2-4ac<0$

$(-5)^2-4\cdot1\cdot(c-1)<0$

$25-4c+4<0$

$29-4c<0$

$29<4c \,/:4$

$c>7{,}25$

zadatke riješio Mladen Sraga

2.4. Vieteove formule

Vietove formule

za:

$$ax^2+bx+c= \begin{cases} x_1+x_2=-\dfrac{b}{a} \\ x_1\cdot x_2=\dfrac{c}{a} \end{cases}$$

Primjeri:

1.) $3x^2-5x+7=0$

$a=3,\ b=-5,\ c=7$

$x_1+x_2=-\dfrac{b}{a}$ $\qquad x_1\cdot x_2=\dfrac{c}{a}$

$x_1+x_2=-\dfrac{-5}{3}$ $\qquad x_1\cdot x_2=\dfrac{7}{3}$

$x_1+x_2=\dfrac{5}{3}$

2.) $x^2-x+110=0$

$a=1,\ b=-1,\ c=110$

$x_1+x_2=-\dfrac{b}{a}$ $\qquad x_1\cdot x_2=\dfrac{c}{a}$

$x_1+x_2=-\dfrac{-1}{1}$ $\qquad x_1\cdot x_2=\dfrac{110}{1}$

$x_1+x_2=1$ $\qquad x_1\cdot x_2=110$

4.) $9x^2+11x-1=0$

$a=9,\ b=11,\ c=-1$

$x_1+x_2=-\dfrac{b}{a}$ $\qquad x_1\cdot x_2=\dfrac{c}{a}$

$x_1+x_2=-\dfrac{11}{9}$ $\qquad x_1\cdot x_2=\dfrac{-1}{9}$

$x_1\cdot x_2=-\dfrac{1}{9}$

5.) $5x^2+x+5=0$

$a=5,\ b=1,\ c=5$

$x_1+x_2=-\dfrac{b}{a}$ $\qquad x_1\cdot x_2=\dfrac{c}{a}$

$x_1+x_2=-\dfrac{1}{5}$ $\qquad x_1\cdot x_2=\dfrac{5}{5}$

$x_1\cdot x_2=1$

1.

1) $2x^2-3x+1=0$

$a=2 \quad b=-3 \quad c=1$

$x_1+x_2=-\frac{b}{a}$ $\qquad$ $x_1\cdot x_2=\frac{c}{a}$

$x_1+x_2=-\frac{-3}{2}$ $\qquad$ $x_1\cdot x_2=\frac{1}{2}$

$x_1+x_2=\frac{3}{2}$

2) $3x^2+x-2=0$

$a=3 \quad b=1 \quad c=-2$

$x_1+x_2=-\frac{b}{a}$ $\qquad$ $x_1\cdot x_2=\frac{c}{a}$

$x_1+x_2=-\frac{1}{3}$ $\qquad$ $x_1\cdot x_2=-\frac{2}{3}$

3) $x^2-x+10=0$

$a=1 \quad b=-1 \quad c=10$

$x_1+x_2=-\frac{b}{a}$ $\qquad$ $x_1\cdot x_2=\frac{c}{a}$

$x_1+x_2=-\frac{-1}{1}$ $\qquad$ $x_1\cdot x_2=\frac{10}{1}$

$x_1+x_2=1$ $\qquad$ $x_1\cdot x_2=10$

4) $2x^2-x-2=0$

$a=2 \quad b=-1 \quad c=-2$

$x_1+x_2=-\frac{b}{a}$ $\qquad$ $x_1\cdot x_2=\frac{c}{a}$

$x_1+x_2=-\frac{-1}{2}$ $\qquad$ $x_1\cdot x_2=\frac{-2}{2}$

$x_1+x_2=\frac{1}{2}$ $\qquad$ $x_1\cdot x_2=-1$

5)

$2x^2+10x-10=0$

$a=2 \quad b=10 \quad c=-10$

$x_1+x_2=-\frac{b}{a}$ $\qquad$ $x_1\cdot x_2=\frac{c}{a}$

$x_1+x_2=-\frac{10}{2}$ $\qquad$ $x_1\cdot x_2=\frac{-10}{2}$

$x_1+x_2=-5$ $\qquad$ $x_1\cdot x_2=-5$

6)

$4x^2+4x+1=0$

$a=4 \quad b=4 \quad c=1$

$x_1+x_2=-\frac{b}{a}$ $\qquad$ $x_1\cdot x_2=\frac{c}{a}$

$x_1+x_2=-\frac{4}{4}$ $\qquad$ $x_1\cdot x_2=\frac{1}{4}$

$x_1+x_2=-1$

zadatke riješio Mladen Sraga

3.

$$3x^2 - x + 2 = 0$$

$$a = 3 \quad b = -1 \quad c = 2$$

1) $$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1 \cdot x_2 = \left(-\frac{b}{a}\right)^2 - 2\frac{c}{a} = \frac{b^2}{a^2} - 2\frac{c}{a} = \frac{b^2}{a^2} - \frac{2c \cdot a}{a \cdot a} = \frac{b^2}{a^2} - \frac{2ac}{a^2} =$$
$$= \frac{b^2 - 2ac}{a^2} = \frac{(-1)^2 - 2 \cdot 3 \cdot 2}{3^2} = \frac{1 - 12}{9} = -\frac{11}{9}$$

2) $$\frac{1}{x_1} + \frac{1}{x_2} = \frac{x_2 + x_1}{x_1 \cdot x_2} = \frac{-\frac{b}{a}}{\frac{c}{a}} = -\frac{b \cdot a}{a \cdot c} = -\frac{b}{c} = -\frac{-1}{2} = \frac{1}{2}$$

3) $$x_1^3 + x_2^3 = (x_1 + x_2)^3 - 3x_1 \cdot x_2 \cdot (x_1 + x_2) = \left(-\frac{b}{a}\right)^3 - 3 \cdot \frac{c}{a} \cdot \left(-\frac{b}{a}\right) = -\frac{b^3}{a^3} + \frac{3bc}{a^2} =$$
$$= \frac{-b^3 + 3abc}{a^3} = \frac{3abc - b^3}{a^3} = \frac{3 \cdot 3 \cdot (-1) \cdot 2 - (-1)^3}{3^3} = \frac{-18 - (-1)}{27} = \frac{-18 + 1}{27} =$$
$$= \frac{-17}{27}$$

4) $$\frac{x_1}{x_2} + \frac{x_2}{x_1} = \frac{x_1 \cdot x_1 + x_2 \cdot x_2}{x_2 \cdot x_1} = \frac{x_1^2 + x_2^2}{x_1 \cdot x_2} = \frac{(x_1 + x_2)^2 - 2x_1 \cdot x_2}{x_1 \cdot x_2} = \frac{\left(-\frac{b}{a}\right)^2 - 2 \cdot \frac{c}{a}}{\frac{c}{a}} =$$
$$= \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c}{a}} = \frac{\frac{b^2 - 2ac}{a^2}}{\frac{c}{a}} = \frac{a(b^2 - 2ac)}{a^2 \cdot c} = \frac{b^2 - 2ac}{ac} =$$
$$= \frac{(-1)^2 - 2 \cdot 3 \cdot 2}{3 \cdot 2} = \frac{1 - 12}{6} = -\frac{11}{6}$$

5) $$x_1^4 + x_2^4 = (x_1^2)^2 + (x_2)^2 = (x_1^2 + x_2^2)^2 - 2x_1^2 \cdot x_2^2 = (x_1^2 + x_2^2)^2 - 2(x_1 \cdot x_2)^2 =$$
$$= \left(\frac{b^2 - 2ac}{a^2}\right)^2 - 2\left(\frac{c}{a}\right)^2 = \left[\frac{(-1)^2 - 2 \cdot 3 \cdot 2}{3^2}\right]^2 - 2 \cdot \left(\frac{2}{3}\right)^2 =$$
$$= \left(\frac{1 - 12}{9}\right)^2 - 2 \cdot \frac{4}{9} = \left(\frac{-11}{9}\right)^2 - \frac{8}{9} = \frac{121}{81} - \frac{8}{9} = \frac{121 - 72}{81} = \frac{49}{81}$$

16.

$$p(x-1)^2 = 2p-1 \qquad p \in R \qquad p \neq 0$$

$$p(x^2-2x+1) = 2p-1$$

$$px^2-2px+p-2p+1=0$$

$$px^2-2px-p+1=0$$

$$a=p \quad b=-2p \quad c=1-p$$

$$x_1+x_2 = -\frac{b}{a} = -\frac{-2p}{p} = 2$$

$$x_1 \cdot x_2 = \frac{c}{a} = \frac{1-p}{p}$$

1) jednadžba ima realne korijene $\Rightarrow D \geq 0$

$$D = b^2-4ac \geq 0$$

$$(-2p)^2-4\cdot p\cdot(1-p) \geq 0$$

$$4p^2-4p+4p^2 \geq 0$$

$$8p^2-4p \geq 0 \,/:4$$

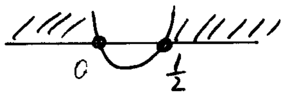
$$2p^2-p \geq 0$$

$$p(2p-1) \geq 0$$

$$p=0 \qquad 2p-1=0$$

$$p_1=0 \qquad 2p=1 \,/:2$$

$$p=\frac{1}{2}$$



$$p \in \langle -\infty, 0] \cup [\tfrac{1}{2}, 0\rangle$$

2) zbroj kubova rješenja jednadžbe jednak je pet

$$x_1^3+x_2^3=5$$

$$x_1^3+x_2^3 = (x_1+x_2)^3-3x_1\cdot x_2(x_1+x_2) = 2^3-3\cdot\frac{1-p}{p}\cdot 2 =$$

$$= 8-6\,\frac{1-p}{p} = 8-\frac{6-6p}{p}$$

$$8-\frac{6-6p}{p} = 5 \,/\cdot p$$

$$8p-(6-6p)=5p$$

$$8p-6+6p=5p$$

$$8p+6p-5p=6$$

$$9p=6 \,/:9$$

$$p=\frac{6}{9} \quad \Rightarrow p=\frac{2}{3}$$

3) razlika rješenja jednadžbe jednaka je 3

$$x_1-x_2=3$$

$$x_1=3+x_2$$

$$x_1+x_2=2$$

$$3+x_2+x_2=2$$

$$3+2x_2=2$$

$$2x_2=2-3$$

$$2x_2=-1 \,/\cdot\frac{1}{2}$$

$$x_2=-\frac{1}{2}$$

$$x_1\cdot x_2=\frac{1-p}{p}$$

$$(3+x_2)\cdot x_2=\frac{1-p}{p}$$

$$\left(3-\frac{1}{2}\right)\cdot\left(\frac{1}{2}\right)=\frac{1-p}{p}$$

$$\frac{6-1}{2}\cdot\left(-\frac{1}{2}\right)=\frac{1-p}{p}$$

$$-\frac{5}{4}=\frac{1-p}{p} \,/\cdot 4p$$

$$-5p=4(1-p)$$

$$-5p=4-4p$$

$$-5p+4p=4$$

$$-p=4 \,/\cdot(-1)$$

$$p=-4$$

zadatke riješio Mladen Sraga

28.

Zadana jednadžba $3x^2 - 2x + 7 = 0$

$a = 3\ ,\ b = -2\ ,\ c = 7$

Rješenja nove jednadžbe označimo sa y_1 i y_2 → zadano je: $y_1 = \dfrac{1}{x_1}\ ,\ y_2 = \dfrac{1}{x_2}$

Nova kvadratna jednadžba je ovog oblika:

$$x^2 - (y_1 + y_2)\cdot x + (y_1 \cdot y_2) = 0$$

$$x^2 - \left(\frac{1}{x_1} + \frac{1}{x_2}\right)\cdot x + \left(\frac{1}{x_1}\cdot\frac{1}{x_2}\right) = 0$$

$$x^2 - \left(\frac{x_2 + x_1}{x_1 \cdot x_2}\right)\cdot x + \left(\frac{1}{x_1 \cdot x_2}\right) = 0 \quad \rightarrow \text{ uvrsti } x_1 + x_2 = -\frac{b}{a}\ ,\ x_1 \cdot x_2 = \frac{c}{a}$$

$$x^2 - \left(\frac{-\frac{b}{a}}{\frac{c}{a}}\right)\cdot x + \frac{1}{\frac{c}{a}} = 0 \quad \rightarrow \text{ pokrati}$$

$$x^2 + \frac{b}{c}\cdot x + \frac{a}{c} = 0$$

$$x^2 + \frac{-2}{3}\cdot x + \frac{7}{3} = 0 \quad /\cdot 3$$

$$3x^2 - 2x + 7 = 0$$

29.

Zadana je jednažba $x^2 - 6x + 5 = 0$

$a = 1\ ,\ b = -6\ ,\ c = 5$

Rješenja nove jednadžbe označimo sa y_1 i y_2 → zadano je: $y_1 = x_1 + 3\ ,\ y_2 = x_2 + 3$

$$y_1 + y_2 = x_1 + 3 + x_2 + 3 = x_1 + x_2 + 6 = -\frac{b}{a} + 6 = -\frac{-6}{1} + 6 = 6 + 6 = 12$$

$$y_1 + y_2 = 12$$

$$y_1 \cdot y_2 = (x_1 + 3)4(x_2 + 3) = x_1x_2 + 3x_1 + 3x_2 + 9 = x_1x_2 + 3(x_1 + x_2) + 9 =$$

$$= \frac{c}{a} + 3\cdot\left(-\frac{b}{a}\right) + 9 = \frac{5}{1} + 3\cdot\left(-\frac{-6}{1}\right) + 9 = 5 + 18 + 9 = 32$$

$$y_1 \cdot y_2 = 32$$

Nova kvadratna jednadžba je ovog oblika:

$$x^2 - (y_1 + y_2)\cdot x + (y_1 \cdot y_2) = 0 \quad \rightarrow \text{ uvrstimo već izračunato: } y_1 + y_2 = 12\ ,\ y_1 \cdot y_2 = 32$$

$$x^2 - 12x + 32 = 0$$