1. KOMPLEKSNI BROJEVI

1.1. Kompleksni broj

1. KOMPLEKSNI BROJ z PIŠEMO:

$$z = a + bi \quad \text{ILI} \quad z = x + yi$$

IMAGINARNA JEDINICA

GDJE JE: $a = Re$, $b = Im$ ILI $x = Re$, $y = Im$

Re – REALNI DIO

Im – IMAGINARNI DIO

i – IMAGINARNA JEDINICA $= \sqrt{-1}$

1) $z = 5 + 2i \rightarrow Re = 5,\ Im = 2$

2) $z = 1 - 3i \rightarrow Re = 1,\ Im = -3$

3) $z = -\frac{1}{2}i \rightarrow Re = 0,\ Im = -\frac{1}{2}$

4) $z = \sqrt{2} \rightarrow Re = \sqrt{2},\ Im = 0$

5) $z = \frac{2-3i}{3} = \frac{2}{3} - \frac{3i}{3} = \underbrace{\frac{2}{3}}_{Re} - \underbrace{i}_{Im = -1} =$ → PAZI IMAGINARNA JEDINICA (i), NE IDE U Im - DIO

$= \frac{2}{3} - 1 \cdot i \rightarrow \underline{\underline{Re = \frac{2}{3}}},\ \underline{\underline{Im = -1}}$

6) $z = \underbrace{1 - \sqrt{2}}_{Re} + i\underbrace{\sqrt{3}}_{Im}$

$Re = 1 - \sqrt{2},\ Im = \sqrt{3}$

U ovom dokumentu imate 12 –stranica sa potpuno riješenim zadacima po školskoj zbirci od autora : Dakić-Elezović – izdavač ELEMENTd.o.o.

Ovo je jako skraćena varijanta naše
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BESPLATNI – PDF – meterijal

Autor rješenja : Mladen Sraga

Matematika-2- KOMPLEKSNI BROJEVI – potpuno riješeni zadaci po školskoj zbirci …

1.2. Zbrajanje i množenje kompleksnih brojeva

1. ZA $z_1 = a+bi$ i $z_2 = c+di$

ZBRAJAMO OVAKO: $z_1 + z_2 = a+bi + c+di = \underline{(a+c)+(b+d)\cdot i}$

ODUZIMAMO OVAKO: $z_1 - z_2 = a+bi-(c+di) = a+bi-c-di =$

$= \underline{(a-c)+(b-d)\cdot i}$

MNOŽIMO OVAKO $z_1 \cdot z_2 = (a+bi)\cdot(c-di) =$

$= \underline{(ac-bd)+(ad+bc)\cdot i}$

MADA KOD MNOŽENJA VEĆINI JE LAKŠE MNOŽITI SVAKI ČLAN PRVE ZAGRADE SA SVAKIM IZ DRUGE ZAGRADE ...

1) $z = -\frac{1}{2}+i$, $w = 1-\frac{1}{3}i$

$z+w = -\frac{1}{2}+i+1-\frac{1}{3}i = -\frac{1}{2}+1+i-\frac{1}{3}i = \frac{-1+2}{2}+\left(1-\frac{1}{3}\right)i =$

$= \underline{\underline{\frac{1}{2}+\frac{2}{3}i}}$

$z-w = -\frac{1}{2}+i-\left(1-\frac{1}{3}i\right) = -\frac{1}{2}+i-1+\frac{1}{3}i =$

$= -\frac{1}{2}-1+i+\frac{1}{3}i = \frac{-1-2}{2}+\left(1+\frac{1}{3}\right)i =$

$= \underline{\underline{-\frac{3}{2}+\frac{4}{3}i}}$

$z\cdot w = \left(-\frac{1}{2}+i\right)\cdot\left(1-\frac{1}{3}i\right) =$

$= -\frac{1}{2}\cdot 1+\left(\left(-\frac{1}{2}\right)\cdot\left(-\frac{1}{3}i\right)\right)+i\cdot 1+i\cdot\left(-\frac{1}{3}i\right) =$

$= -\frac{1}{2}+\frac{1}{6}i+i-\frac{1}{3}i^2 =$

$= -\frac{1}{2}+\left(\frac{1}{6}+1\right)i-\frac{1}{3}\cdot(-1) =$

$= -\frac{1}{2}+\frac{1}{3}+\frac{7}{6}i$

$= \underline{\underline{-\frac{1}{6}+\frac{7}{6}i}}$

$i = \sqrt{-1}$

$i^2 = \sqrt{-1}^{\,2} = -1$

$i^3 = i^2\cdot i^1 = -1\cdot i = -i$

$i^4 = i^2\cdot i^2 = (-1)\cdot(-1) = 1$

$i^5 = i^4\cdot i^1 = 1\cdot i = i$

ITD.

1.2.

2) $z = -2+3i \quad , \quad w = 2+i$

$z+w = -2+3i+2+i = -2+2+3i+i = 0+(3+1)i = \underline{\underline{4i}}$

$z-w = (-2+3i)-(2+i) = -2+3i-2-i = -2-2+3i-i =$

$= -4+(3-1)i = \underline{\underline{-4+2i}}$

$z \cdot w = (-2+3i)\cdot(2+i) = -2\cdot 2+(-2)\cdot i+3i\cdot 2+3i\cdot i =$

$= -4-2i+6i+3i^2 =$

$= -4+4i+3\cdot(-1) =$

$= -4+4i-3 =$

$= \underline{\underline{-7+4i}}$

3) $z = \frac{3}{4}-\frac{2}{3}i \quad , \quad w = \frac{3}{4}+\frac{1}{3}i$

$z+w = \frac{3}{4}-\frac{2}{3}i+\frac{3}{4}+\frac{1}{3}i = \frac{3}{4}+\frac{3}{4}-\frac{2}{3}i+\frac{1}{3}i =$

$= \frac{6}{4}+\left(-\frac{2}{3}+\frac{1}{3}\right)i =$

$= \underline{\underline{\frac{3}{2}-\frac{1}{3}i}}$

$z-w = \frac{3}{4}-\frac{2}{3}i-\left(\frac{3}{4}+\frac{1}{3}i\right) = \frac{3}{4}-\frac{2}{3}i-\frac{3}{4}-\frac{1}{3}i =$

$= \frac{3}{4}-\frac{3}{4}+\left(-\frac{2}{3}-\frac{1}{3}\right)\cdot i =$

$= 0+\left(-\frac{3}{3}\right)\cdot i$

$= 0+-1i = \underline{\underline{-i}}$

$z \cdot w = \left(\frac{3}{4}-\frac{2}{3}i\right)\cdot\left(\frac{3}{4}+\frac{1}{3}i\right) =$

$= \frac{3}{4}\cdot\frac{3}{4}+\frac{3}{4}\cdot\frac{1}{3}i-\frac{2}{3}i\cdot\frac{3}{4}-\frac{2}{3}i\cdot\frac{1}{3}i =$

$= \frac{9}{16}+\frac{1}{4}i-\frac{1}{2}i-\frac{2}{9}i^2 = \frac{9}{16}+\left(\frac{1}{4}-\frac{1}{2}\right)i-\frac{2}{9}\cdot(-1) = \frac{9}{16}-\frac{1}{4}i+\frac{2}{9} =$

$= \underline{\underline{\frac{113}{144}-\frac{1}{4}i}}$

1.2.

16.

1.) $(1-i)\cdot x + (1+i)y = i$

$x - xi + y + yi = i$

$x + y - xi + yi = i$

$x + y + (-x+y)i = 0 + i$

$x+y \to Re_1$, $(-x+y) \to Im_1 = -x+y$, $0 \to Re_2$, $i \to Im_2 = 1$

MORA BITI $Re_1 = Re_2$ i $Im_1 = Im_2$

$\underbrace{x + y = 0 \qquad -x + y = 1}_{\text{RIJEŠIMO SUSTAV}}$

$$\left.\begin{aligned} x+y &= 0 \\ -x+y &= 1 \end{aligned}\right\} +$$

$2y = 1 \quad /:2$

$y = \frac{1}{2}$

$x + y = 0$

$x + \frac{1}{2} = 0$

$x = -\frac{1}{2}$

2.) $(2-3i)x - (1+4i)y = 3+i$

$2x - 3xi - (y + 4yi) = 3 + i$

$2x - 3xi - y - 4yi = 3 + i$

$2x - y - 3xi - 4yi = 3 + i$

$2x - y + (-3x - 4y)i = 3 + i$

$2x-y \to Re_1$, $(-3x-4y) \to Im_1$, $3 \to Re_2$, $i \to Im_2 = 1$

$Re_1 = Re_2 \qquad Im_1 = Im_2$

$\underbrace{2x - y = 3 \qquad -3x - 4y = 1}_{\text{SUSTAV}}$

$2x - y = 3 \quad /\cdot(-4)$

$-3x - 4y = 1$

$$\left.\begin{aligned} -8x + 4y &= -12 \\ -3x - 4y &= 1 \end{aligned}\right\} +$$

$-11x = -11 \quad /:-1$

$x = 1$

$x = 1$

$2x - y = 3$

$2\cdot 1 - y = 3$

$-y = 3 - 2$

$-y = 1 \quad /\cdot(-1)$

$y = -1$

21.

1.2.

U ovom zadatku koristimo pravilo $i^{4\cdot m} = 1$ jer je $i^{4\cdot m} = (i^4)^m = 1^m = 1$ za $m \in Z$

br.1	$i^{4\cdot m}$	$= 1$
br.2	$i^{4\cdot m+1}$	$= i^{4\cdot m}\cdot i^1 = 1\cdot i = i$
br.3	$i^{4\cdot m+2}$	$= i^{4\cdot m}\cdot i^2 = 1\cdot(-1) = -1$
br.4	$i^{4\cdot m+3}$	$= i^{4\cdot m}\cdot i^3 = 1\cdot(-i) = -i$

zapamti $\begin{cases} i^1 = i \\ i^2 = -1 \\ i^3 = -i \\ i^4 = 1 \end{cases}$

PRIMJERI

1.) $i^{15} =$ | sada eksponent 15 rastavimo na najveći broj djeljiv sa 4 i ostatak (15 : 4 = 3 i ostatak je 3) pa eksponent 15 pišemo 15 = 4 · 3 + 3 sada je to pravilo br.4

4 · m + 3 to nam govori da radimo po pravilu br.4

$i^{15} = i^{4\cdot 3+3} = i^{4\cdot 3}\cdot i^3 = 1\cdot(-i) = -i$

4 · m + 1 to nam govori da radimo po br.2

2.) $i^{505} = i^{4\cdot 126+1} = i^{4\cdot 126}\cdot i^1 = 1\cdot i = i$

4 · m – to je br.1

3.) $i^{100000} = i^{4\cdot 25000} = 1$

4.) $i^{12345} = i^{4\cdot 3086+1} = i^{4\cdot 3086}\cdot i^1 = 1\cdot i = i$

5.) $i^{54321} = i^{4\cdot 13580+1} = i^{4\cdot 13580}\cdot i^1 = i$

6.) $i^{200002} = i^{4\cdot 50000+2} = i^{4\cdot 50000}\cdot i^2 = 1\cdot(-1) = -1$

1.24. 1) $i^{77} = i^{4\cdot 19+1} = i^{4\cdot 19}\cdot i^1 = 1\cdot i = \underline{\underline{i}}$

$i^{4\cdot 19} = (i^4)^{19} = 1^{19} = \underline{\underline{1}} \rightarrow$ PO PRAVILU BR. 1.

$77 = 4\cdot 19 + 1$

2) $i^{2468} = i^{4\cdot 617+0} = (i^4)^{617} = 1^{617} = \underline{\underline{1}}$

3) $i^{3579} = i^{4\cdot 894+3} = i^{4\cdot 894}\cdot i^3 = 1\cdot i^3 = 1\cdot(-i) = \underline{\underline{-i}}$

4) $(i^{111})^{33} = i^{3663} = i^{4\cdot 915+3} = i^{4\cdot 915}\cdot i^3 = 1\cdot i^3 = 1\cdot(-i) = \underline{\underline{-i}}$

1.2.

22.

1)

$$1+i^3+i^6+i^9+i^{12}+i^{15}=$$

$$=1+(-i)+i^{4+2}+i^{2\cdot4+1}+i^{3\cdot4}+i^{3\cdot4+3}$$

$$=1-i+i^2+i+1+i^3$$

$$=2-1+(-i)$$

$$=1-i$$

2)

PRVO IZRAČUNAMO:

$$\begin{aligned}
i^{111} &= i^{27\cdot4+3} = i^3 = -i\\
i^{222} &= i^{55\cdot4+2} = i^2 = -1\\
i^{333} &= i^{83\cdot4+1} = i^1 = i\\
i^{444} &= i^{4\cdot111} = 1^{111} = 1\\
i^{555} &= i^{4\cdot138+3} = i^3 = -i\\
i^{666} &= i^{4\cdot166+2} = i^2 = -1\\
i^{777} &= i^{4\cdot194+1} = i^1 = i\\
i^{888} &= i^{4\cdot222} = 1^{222} = 1\\
i^{999} &= i^{4\cdot249+3} = i^3 = -i
\end{aligned}$$

$$i^{111}+i^{222}+i^{333}+\dots i^{999} = \rightarrow$$

= UVRSTIMO DOBIVENE VRIJEDNOSTI

$$=-i-1+i+1-i-1+i+1-i=$$

$$=-i+i-i+i-i-1+1-1+1=$$

$$=-i+0=$$

$$=-i$$

1.3. Dijeljenje kompleksnog broja

1.

$Z = a + bi$ $\quad \bar{Z} = a - bi$ konjugirano kompleksni broj broja Z $\quad$ a = realni dio, b = imaginarni dio $\}$ kompleksnog broja Z

kako se vidi konjugirano kompleksni broj broja Z razlikuje se od Z u tome što imaginarni dio ima suprotni predznak od imaginarnog djela broja Z

PRIMJERI:

1.) $z = -1 + 3i \rightarrow \bar{z} = -1 - 3i$ imaginarni dio broja z je 3 pa imaginarni dio konjugirano kompleksnog broja broja z je -3 (dakle samo suprotni predznak)

2.) $z = \frac{1}{2} - \frac{4}{3}i \rightarrow \bar{z} = \frac{1}{2} + \frac{4}{3}i$

3.) $z = -2i \rightarrow \bar{z} = 2i$

4.) $z = 1 \rightarrow \bar{z} = 1$ jer je Im od $z = 0$ pa je je i Im od $\bar{z} = 0$

5.) $z = 1 - 112i \rightarrow \bar{z} = 1 + 112i$

6.) $z = -\sqrt{313} \rightarrow \bar{z} = -\sqrt{313}$ isto kao i 4.)

1) $z = -1 + 3i \longrightarrow \bar{z} = -1 - 3i$

2) $z = 1 + \sqrt{22} \qquad \bar{z} = 1 + \sqrt{22}$
jer je Jm od $z = 0$ pa je i
Jm od $\bar{z} = 0$

3) $z = \frac{1-\sqrt{2}}{3}i \longrightarrow \bar{z} = -\frac{1-\sqrt{2}}{3}i$

$$\bar{z} = \frac{-(1-\sqrt{2})}{3}i$$

$$\bar{z} = \frac{-1+\sqrt{2}}{3}i$$

4) $z = 1 - (\sqrt{2} - \sqrt{3})i \longrightarrow \bar{z} = 1 + (\sqrt{2} - \sqrt{3})i$

5) $z = 1 - \sqrt{2} + \sqrt{3}i \longrightarrow \bar{z} = 1 - \sqrt{2} - \sqrt{3}i$

2.

$$z = \underbrace{2m+n}_{Re} + \underbrace{m}_{Jm}i \qquad w = \underbrace{m}_{Re} - \underbrace{(n-3)}_{Jm}i$$

Da bi kompleksni brojevi z i w međusobno bili kompleksno konjugirani moraju imati jednake realne dijelove $Re_z = Re_w$ a imaginarni djelovi moraju biti suprotnog predznaka $Jm_z = -Jm_w$

$$Re_z = Re_w \qquad\qquad Jm_z = -Jm_w$$
$$2m + n = m \qquad\qquad m = -[-(n-3)]$$
$$2m - m + n = 0 \qquad\qquad m = n - 3$$
$$m + n = 0 \qquad\qquad m - n = -3$$

metodom suprotnih koeficijenata nađemo realne brojeve m i n.

4.

1.3.

Dijeljenje kompleksnih brojeva $\frac{a+bi}{c+di}$ izvodimo tako da razlomak pomnožimo sa $\frac{c-di}{c-di}$ tj. i brojnik i nazivnik pomnožimo sa konjugiranim nazivnikom

PRIMJERI

1.) $\frac{i}{1+i} = \frac{i}{1+i} \cdot \frac{1-i}{1-i} = \frac{i\cdot(1-i)}{1^2-i^2} = \frac{i-i^2}{1-(-1)} = \frac{i-(-1)}{1+1} = \frac{i+1}{2} = \frac{i}{2}+\frac{1}{2} = \frac{1}{2}+\frac{1}{2}i$

2.) $\frac{1-i}{i} = \frac{1-i}{i}\cdot\frac{-i}{-i} = \frac{(1-i)\cdot(-i)}{i\cdot(-i)} = \frac{-i+i^2}{-i^2} = \frac{-i-1}{-(-1)} = \frac{-1-i}{1} = -1-i$

3.) $\frac{1+i}{1-i} = \frac{1+i}{1-i}\cdot\frac{1+i}{1+i} = \frac{(1+i)^2}{1^2-i^2} = \frac{1+2i+i^2}{1-(-1)} = \frac{1+2i-1}{1+1} = \frac{2i}{2} = i$

4.) $\frac{-2+i}{1+3i} = \frac{-2+i}{1+3i}\cdot\frac{1-3i}{1-3i} = \frac{-2+6i+i-3i^2}{1^2-(3i)^2} = \frac{-2+7i-3\cdot(-1)}{1-9\cdot i^2} = \frac{-2+7i+3}{1-9\cdot(-1)} = \frac{1+7i}{1+9} = \frac{1}{10}+\frac{7}{10}i$

5.) $\frac{-1+31i}{-1+5i} = \frac{-1+31i}{-1+5i}\cdot\frac{-1-5i}{-1-5i} = \frac{1+5i-31i-155i^2}{(-1)^2-(5i)^2} = \frac{1-26i-155\cdot(-1)}{1-25\cdot i^2} = \frac{1-26i+155}{1-25\cdot(-1)} =$

$= \frac{156-26i}{1+25} = \frac{156}{26}-\frac{26i}{26} = 6-i$

6.) $\frac{5+5i}{3+i} = \frac{5+5i}{3+i}\cdot\frac{3-i}{3-i} = \frac{15-5i+15i-5i^2}{3^2-i^2} = \frac{15+10i-5\cdot(-1)}{9-(-1)} = \frac{15+10i+5}{9+1} = \frac{20+10i}{10} = \frac{20}{10}+\frac{10i}{10} =$

$= 2+i$

zadaci:

1) $\frac{3-i}{1+i} = \frac{3-i}{1+i}\cdot\frac{1-i}{1-i} = \frac{3-3i-i+i^2}{1^2-i^2} = \frac{3-4i-1}{1+1} = \frac{2-4i}{2} =$

$= \frac{2(1-2i)}{2} = 1-2i$

2) $\frac{1+2i}{1-i} = \frac{1+2i}{1-i}\cdot\frac{1+i}{1+i} = \frac{1+i+2i+2i^2}{1^2-i^2} = \frac{1+3i-2}{1+1} = \frac{-1+3i}{2} =$

$= -\frac{1}{2}+\frac{3}{2}i$

3) $\frac{-4i}{\sqrt{3}+i} = \frac{-4i}{\sqrt{3}+i}\cdot\frac{\sqrt{3}-i}{\sqrt{3}-i} = \frac{-4i\sqrt{3}+4i^2}{(\sqrt{3})^2-i^2} = \frac{-4i\sqrt{3}-4}{3+1} =$

$= \frac{4(-i\sqrt{3}-1)}{4} = -i\sqrt{3}-1 = -1-i\sqrt{3}$

4) $\frac{1+i\sqrt{3}}{\sqrt{3}-i} = \frac{1+i\sqrt{3}}{\sqrt{3}-i}\cdot\frac{\sqrt{3}+i}{\sqrt{3}+i} = \frac{\sqrt{3}+i+i(\sqrt{3})^2+i^2\sqrt{3}}{(\sqrt{3})^2-i^2} = \frac{\sqrt{3}+i+3i-\sqrt{3}}{3+1} =$

$= \frac{4i}{4} = i$

5) $\frac{1-i}{i} = \frac{1-i}{i}\cdot\frac{-i}{-i} = \frac{-i+i^2}{-i^2} = \frac{-i-1}{-(-1)} = \frac{-1-i}{1} = -1-i$

1.3.

10.

1) za $z = -1 + 2i$ $\quad w = 2 - 3i$

$\bar{z} = -1 - 2i$ $\quad \bar{w} = 2 + 3i$

$$\frac{z\bar{w} - \bar{z}w}{z^2 - w^2} = \frac{(-1+2i)\cdot(2+3i) - (-1-2i)(2-3i)}{(-1+2i)^2 - (2-3i)^2} =$$

$$= \frac{-2-3i+4i+6i^2 - (-2+3i-4i+6i^2)}{(-1)^2 - 4i + 2^2i^2 - (2^2 - 12i + 3^2i^2)} =$$

$$= \frac{-2+i-6-(-2-i-6)}{1-4i-4-(4-12i-9)} =$$

$$= \frac{-8+i+2+i+6}{-3-4i-4+12i+9} = \frac{2i}{2+8i} = \frac{2i}{2(1+4i)} = \frac{i}{1+4i} =$$

$$= \frac{i}{1+4i} \cdot \frac{1-4i}{1-4i} = \frac{i-4i^2}{1^2 - 4^2i^2} = \frac{i+4}{1+16} = \frac{4+i}{17} =$$

$$= \frac{4}{17} + \frac{1}{17}i$$

1.3.

12. $z = \frac{i^{357}}{(1-2i)(3+i)}$, $Re\, z = ?$

$$z = \frac{i^{357}}{(1-2i)(3+i)} = \frac{i^{4\cdot 89+1}}{3+i-6i-2i^2} = \frac{i^1}{3-5i-2\cdot(-1)} =$$

$$= \frac{i}{3-5i+2} = \frac{i}{5-5i} =$$

$$= \frac{i}{5-5i} \cdot \frac{5+5i}{5+5i} = \frac{5i+5i^2}{5^2-5^2 i^2} =$$

$$= \frac{5i+5\cdot(-1)}{25-25\cdot(-1)} = \frac{5i-5}{25+25} = \frac{-5+5i}{50} =$$

$$= -\frac{5}{50} + \frac{5}{50} i = -\frac{1}{10} + \frac{1}{10} i$$

$$\downarrow$$

$$Re\, z = -\frac{1}{10}$$

13. $Im\, z = ?$

$$z = \frac{i^{246}}{(1+2i)(3-i)} = \frac{i^{4\cdot 61+2}}{3-i+6i-2i^2} = \frac{i^2}{3+5i-2\cdot(-1)} =$$

$$= \frac{-1}{3+5i+2} = \frac{-1}{5+5i} =$$

$$= \frac{-1}{5+5i} \cdot \frac{5-5i}{5-5i} =$$

$$= \frac{-5+5i}{5^2-5^2 i^2} = \frac{-5+5i}{25-25\cdot(-1)} =$$

$$= \frac{-5+5i}{25+25} = -\frac{5}{50} + \frac{5}{50} i =$$

$$= -\frac{1}{10} + \frac{1}{10} i$$

$$\downarrow$$

$$Im\, z = \frac{1}{10}$$

1.3.

19.

1) $\left(\dfrac{i^{55}-i^{66}}{i^{77}+i^{88}}\right)^{99} = \left|\begin{array}{l} i^{55} = i^{13\cdot4+3} = i^3 = -i \\ i^{66} = i^{16\cdot4+2} = i^2 = -1 \\ i^{77} = i^{19\cdot4+1} = i^1 = i \\ i^{88} = i^{22\cdot4} = i^0 = 1 \end{array}\right| =$

$$= \left(\frac{-i-(-1)}{i+1}\right)^{99} = \left(\frac{-i+1}{1+i}\right)^{99} = \left(\frac{1-i}{1+i}\cdot\frac{1-i}{1-i}\right)^{99} =$$

$$= \left(\frac{(1-i)^2}{1^2-i^2}\right)^{99} = \left(\frac{1^2-2i+i^2}{1+1}\right)^{99} = \left(\frac{1-2i-1}{2}\right)^{99} =$$

$$= \left(\frac{-2i}{2}\right)^{99} = (-i)^{99} = (-1)^{99}\cdot(i)^{99} = -i^{24\cdot4+3} = -i^3 =$$

$$= -(-i) = i$$

2) $\left(i^{101}+\dfrac{i^{202}}{i^{303}}\right)^{404} = \left|\begin{array}{l} i^{101} = i^{25\cdot4+1} = i^1 = i \\ i^{202} = i^{50\cdot4+2} = i^2 = -1 \\ i^{303} = i^{75\cdot4+3} = i^3 = -i \end{array}\right| =$

$$= \left(i+\frac{-1}{-i}\right)^{404} = \left(i+\frac{1}{i}\right)^{404} = \left(\frac{i^2+1}{i}\right)^{404} = \left(\frac{-1+1}{i}\right)^{404} = \left(\frac{0}{i}\right)^{404} = 0$$

3) $\left(\dfrac{i^{55}-1}{1+i^{55}}\right)^{55} = \left| i^{55} = i^{13\cdot4+3} = i^3 = -i \right| = \left(\dfrac{(-i)-1}{1+(-i)}\right)^{55} = \left(\dfrac{-1-i}{1-i}\right)^{55} =$

$$= \left[\frac{-(1+i)}{1-i}\cdot\frac{(1+i)}{1+i}\right]^{55} = \left[\frac{-(1+i)^2}{1^2-i^2}\right]^{55} = \left[\frac{-(1+2i+i^2)}{1+1}\right]^{55} =$$

$$= \left[\frac{-(1+2i-1)}{2}\right]^{55} = \left[\frac{-(+2i)}{2}\right]^{55} = \left[\frac{-2i}{2}\right]^{55} = (-i)^{55} = (+i)^{55}\cdot(-1)^{55} = -(-i) = i$$